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CHANGES IN THE WAVE-FREQUENCIES OF THE LINES OF EMISSION SPECTRA OF ELEMENTS, THEIR DEPENDENCE UPON THE ELEMENTS THEMSELVES AND UPON THE PHYSICAL CONDITIONS UNDER WHICH THEY ARE PRODUCED.

By W. J. HUMPHREYS.

PRELIMINARY REMARKS.

IT is well known to spectroscopists that the character of the emission spectrum of an element depends greatly upon the physical conditions under which it is produced. If the element is in the solid or liquid state its spectrum is continuous, but discontinuous when it is in the form of an attenuated gas. The discontinuous spectrum may consist either of bands or of isolated lines or of both, according in part to the substance used, and in part to the conditions under which its spectrum is formed. In general the number of lines that can be detected and their intensities increase with increase of temperature, though their relative intensities may change very greatly as the temperature is raised. Increase in the density of a gas or vapor increases the width of its spectral lines; some of them spread out symmetrically, others unsymmetrically. In the latter case the chief increase in width is usually, but not always, towards the less refrangible or red

end of the spectrum. A somewhat similar increase in the width of lines (both emission and absorption), together with certain polarization phenomena, may be produced, according to Zeeman,[1] by placing the vapor to which the lines are due in a strong magnetic field.

Again, whether a line is reversed or not depends in part at least upon the thickness and density of the absorbing layer. Finally a single element, as argon for instance, may give one or another of two distinct line spectra owing to the character of the electric discharge used to produce it, and to the pressure of the gas.

Not only emission but absorption spectra also are known to be subject to changes. One of the most important of these changes was observed by Kundt who, in describing it, says that the position of an absorption band depends upon the substance in which it is dissolved or incorporated, that the band is displaced towards the red end of the spectrum when the substance producing it is dissolved in a strongly dispersive medium; or, to use his own words, as they occur in an article on absorption spectra.[2] "Hat ein farbloses Lösungsmittel ein beträchtlich grösseres Brechungs-und Dispersionsvermögen als ein anderes, so liegen die Absorptionsstreifen einer in den Medien gelösten Substanz bei Anwendung des ersten Mittels dem roten Ende des Spectrums näher als bei Benutzung des zweiten."

In view of the reciprocal relation between absorption and emission of radiations, it would seem that one might suspect the possibility of a similar phenomenon, that is shift of lines, in the case of emission spectra. Indeed some observers have reported shifts of certain spectral lines, and besides something of the kind is suggested by the theories of both Lommel[3] and Wüllner.[4]

[1] "On the influence of Magnetization on the Nature of the Light Emitted by a Substance."—*Phil. Mag.*, March 1897; this JOURNAL, **5**, 332, 1897.

[2] "Ueber den Einfluss des Lösungsmittels auf die Absorptionsspectra gelöster absorbirender Medien."—*Wied. Ann.*, **4**, 34.

[3] "Theorie der Absorption und Fluorescenz."—*Wied. Ann.*, **3**, 251.

[4] "Ueber almähliche Ueberführung des Bandenspectrums des Stickstoffs in ein Linienspectrum."—*Wied. Ann.*, **8**, 590.

However, these observations were all but certainly illusive, as others have asserted, and as will be explained further on, and the theories are at least incomplete since they do not agree in all respects with observations. In considering them I shall confine myself to those parts that deal with the displacements or shifts of the lines.

According to Lommel's theory the spectral lines increase in width, chiefly on the red side, and shift in the same direction when the density of the gas producing them is increased. He says: "Bei vergrösserung der Dichte oder des Drucks eines Gases erleidet die helle Spectrallinie eine Verbreiterung und gleichzeitige Verschiebung nach der weniger brechbaren Seite hin."

This theory makes the spreading of a line and its displacement depend upon the same thing—namely, the increase of the density of the gas whose lines are affected. According to it the spreading must always be chiefly towards the red end of the spectrum, and the lines must shift only when they are spread out; in fact the shift of a line is due, in terms of the theory, to unsymmetrical broadening and to nothing else.

As a matter of fact, while many lines are spread out, by increase of the density of the gas producing them, chiefly towards the red end of the spectrum, many others are broadened symmetrically, and others even spread out chiefly towards the more refrangible or violet end. Therefore by merely increasing the density of the luminous gas or vapor, without change of total pressure, the centers of certain lines are moved toward the red and others toward the violet of the spectrum; while those lines that spread symmetrically are not displaced at all. Besides the reversals, which of course give the positions of the most intense portions of the lines, are never displaced in the slightest by any increase in the density of the luminous gas or vapor so long as the absolute pressure is kept the same.

The shift discussed in this paper probably does not depend in the least, as will appear from the experimental results, upon the density of the gas or vapor producing the lines, but only

upon the absolute pressure; and apparently it has no connection with the spreading, either symmetrical or unsymmetrical, of the lines themselves. It would therefore seem that Lommel's theory, though ingenious and well worked out, in no wise predicts the observations described in the following pages.

In support of that part of his theory which demands a shift of the lines, Lommel refers to the experiments of Zöllner[1] and Müller,[2] both of whom used the common method of putting a bead of salt in a Bunsen flame and then examining, by suitable methods, the light so produced. The intensity of the flame and the quantity of salt in it were both varied, and some of the results they obtained indicated a movement of the lines towards the red end of the spectrum. This can be explained by, and was almost certainly due to, unsymmetrical spreading of the lines examined. Certainly neither observer found (the conditions were not such as would produce it) a true displacement of the lines in the sense that the term is used in this paper. That is, they did not find a given line, produced under certain conditions, differing from the same line when produced under other conditions in any wise except mere position in the spectrum; nor do they speak of the displacement of the reversals.

The other theory referred to, that of Wüllner, assumes all variations in the spectrum of a substance to be due to what might be termed external changes, such as temperature, density, and the like, and in no case, not even in the change from band to isolated line spectra, to any alteration of molecular grouping. Wüllner assumes the correctness of Zöllner's equation,

$$E = \left\{ 1 - (1 - a)^{d\delta} \right\} e,$$

in which E is the total amount of light of a given wave-length, d the thickness, δ the density, a the coefficient of absortion of the luminous gas or vapor, and e the power of a perfectly black

[1] "Ueber den Einflus der Dichtigkeit und Temperatur auf die Spectra glühender Gase." *Pogg. Ann.*, **142**, 88.

[2] "Beobachtungen über die Interferenz des Lichtes bei grossen Gangunterschieden." *Pogg. Ann.*, **150**, 311.

body, at the same temperature as the luminous gas, to give out light of the given wave-length. If *a* is a function not only of wave-length but of temperature too, and such a function of them that its maximum value occurs at different places for different temperatures, as Wüllner assumes it to be, then clearly a line may be shifted by merely changing the temperature of its source. Besides, the shift may be in either direction and may be regular or irregular. In short, if *a* is such a function of temperature and of wave-length as that just described one can only say that a given change in temperature will produce a greater or less change, in one direction or the other, in the position of a line in the spectrum. In certain respects the conclusions of this theory are not supported by careful observations, and for this reason it is not now, if ever, well received by spectroscopists. In regard to the displacements of the lines it is stated by Kayser,[1] in an article in which he discusses the above theory quite fully, that he knows of only one line, D_2, of which accurate measurements have indicated a shift, and that in this case the shift is illusive, and due to unsymmetrical broadening. His words are: "Mir ist nur ein Fall bekannt wo man nach genauen Messungen eine Verschiebung glaubte beobachten zu können: nämlich bei der Linie D_2; aber dies ist, wie an anderer Stelle gezeigt werden soll, eine Täuschung: D_2 verbreitet sich nicht gleichmässig nach beiden Seiten, daher scheint sich die Mitte etwas zu verschieben, aber der hellste Theil, die eigentliche Linie bleibt genau an ihrer Stelle." In another place[2] Kayser says that neither lines nor bands have ever been observed to shift. "Eine Verschiebung von Linien ist selbst von den genauesten Messungen niemals beobachtet worden, weder beim Banden- noch beim Linienspectrum."

Before the present work was begun no accurate experiments, so far as I can learn, had shown a true shift independent of all other changes of the spectral lines, nor had it been demanded by theory. Lommell's theory made the shifts of the lines a conse-

[1] "Ueber den Ursprung des Banden- und Linien Spectrums." *Wied. Ann.*, **42**, 310.

[2] WINKELMANN, *Handbuch der Physik*, II, 1, p. 425.

quence of their unsymmetrical spreading, while Wüllner's theory, though capable of explaining any one of several phenomena, is too flexible to predict shifts of lines at all definitely. It only states in this particular what is perfectly evident, namely, that by changing the conditions under which the spectrum of a substance is produced its lines may or may not be displaced, which of course is really predicting nothing.

As stated above, neither of these theories has been supported as to the shifts of the lines by observations. Instead then of regarding the wave-frequency of a line, and consequently its wave-length and position in the spectrum, as being one thing or another, owing to circumstances, practically all spectroscopists have considered it a constant of reference (except as modified by the Doppler effect), subject to no possible change. All observers who have given us tables of accurately determined wave-lengths have, at least tacitly, made this assumption, and upon it are based the estimates of the velocities in the line of sight of many of the fixed stars. The same assumption is made in comparing solar and stellar with terrestrial spectra for the purpose of determining the constituents of the Sun and stars, and when the coincidence of lines evidently the same was not exact the discrepancy was naturally referred to some cause other than actual change in wave-length.

This assumption of the constancy of wave-frequency has led to the hope, a vain one it seems, that wave-lengths of spectral lines may serve as ideal units of reference—units whose values are absolutely the same at all times and under all circumstances.

While the wave-frequencies of spectral lines depend, as shown further on in this paper, upon the physical conditions under which they are produced and therefore their wave-lengths are not ideal units of reference, still it is easy to obtain spectral lines, as often as desired, under conditions so similar that their wave-lengths are more nearly ideal length units than are those which we can at present obtain in any other way. Consequently the results of the experiments described in the following pages do not materially affect (though they show precautions that must

be taken) the value of Professor Michelson's[1] most ingenious and careful determination of the wave-lengths of the red, green, and blue lines of the spark-spectrum of cadmium vapor at low pressure, in terms of the standard meter.

OBJECT OF THE INVESTIGATION.

The work described in this paper was suggested by Dr. Ames and begun for the purpose of examining minutely the effect of pressure on the arc spectra of various elements, and in particular for noting the effect if any on the wave-lengths of the lines. The idea of examining arc spectra under pressure occurred to Professor Rowland several years ago, and the apparatus used in the present investigation is that which he had constructed for this purpose. Constant work however along other lines prevented him from making any observations with it.

The first accurate observations, to the best of my knowledge, that suggested the probability of a functional relation between the wave-frequencies of spectral lines and the conditions under which the lines are produced were made by Mr. L. E. Jewell in the physical laboratory of the Johns Hopkins University. The suggestion came in part from the fact that Mr. Jewell's numerous and careful measurements of the same lines in the arc and solar spectra showed a want of coincidence which varied for different elements. While this want of coincidence was never great, still it seemed too regular to admit of the apparently obvious explanation that it was not due to any real difference in wave-length, but to some disturbance of the apparatus during the exposure of the photographic plate. Mr. Jewell had also obtained slight real or apparent displacements of certain lines by changing the amount of material in the arc. It was this chiefly that led Dr. Ames to suggest the present investigation.

The only way, of course, to determine whether such a functional relation between wave-lengths of spectral lines and the conditions under which the spectra are produced actually exists,

[1] "Determination experimental de la valeur du mètre en longueurs d'ondes lumineuses."

was by direct experiment, and it therefore seemed advisable to examine arc spectra under different conditions, especially of pressure and so far as possible of temperature too, since the conditions under which solar and ordinary spectra are produced may differ greatly in both these respects.

Another reason for taking up this investigation was found in the fact that the wave-lengths of the red, blue, and green cadmium lines as determined by Professor Michelson for the purpose of accurately comparing them with the standard meter, were less in each case than those of the same lines as determined by Professor Rowland. These differences are .208 of an Ångström unit for the red, .173 for the green, and .186 for the blue line. In each case the difference amounts to only about one part of the wave-length in thirty thousand, and in itself is not very surprising, since the methods followed by these two able men were totally different; but it is rather surprising that these differences are not constant. Here again differences of physical conditions under which the spectra were produced (Michelson worked with spark spectra at low pressure, while Rowland used the arc at atmospheric pressure) suggest a possible explanation of the want of agreement in their measurements. The results of numerous examinations of cadmium spectra as produced under different pressures do account for a part, though only about 5 per cent., of the above differences, but fail to suggest, at least very definitely, why the differences should not be the same for all the lines.

APPARATUS USED.

The grating used in all this work was a six-inch Rowland concave of twenty-one and a half feet focal length and ruled with 20,000 lines to the inch. It was mounted in the usual way, as fully described by Dr. Ames in the Johns Hopkins *Circular* of May 1889. The arc was produced by a direct 110-volt current of any amperage desired. It was found necessary to make the strength of the current very different for different substances and also for different amounts in the arc of the same substance. At times the current was small — only a few amperes—while occasion

ally, judging from the fuses blown and other effects, it was little if any less than one hundred. The effect of varying the strength of the current will be discussed further on. The poles of the electric arc were used vertical and parallel to the slit of the spectroscope. Horizontally mounted poles were also tested, and the results will be given in the proper place, but it did not appear necessary to use any other than the vertical mounting, which was found to be much the more convenient of the two tried.

A number of photographs were taken of the spectrum given by an arc between one carbon and one (the lower) metallic pole, and a few of the spectrum formed by an arc between two metallic poles. The metals so used were iron, copper, brass and zinc. In all other cases both poles were of carbon, the lower one being bored axially to a depth of from one to three inches. This cavity, which was about an eighth of an inch in diameter, was filled with the substance or substances whose spectrum was desired. Very often elements were used in the metallic form, but as a rule it was more convenient and occasionally, as in the case of sodium and potassium, much better to use some compound. The quantity of the element or compound in the arc could easily be reduced, as was often necessary, by mixing it with carbon dust before charging the pole with it. In nearly all casest he pole carrying the charge was made the positive one.

The pressure around the arc was obtained in every instance by pumping air into the apparatus designed for this work by Professor Rowland, as stated above, and used by Messrs. Duncan, Rowland and Todd[1] in their examination of the electric arc under pressure. The structure of this apparatus may be understood by aid of the accompanying sketch, Fig. 1, in which *A* is an iron cylinder fourteen inches high and seven inches in diameter. *B*, *B* are suitably constructed stuffing boxes through which the rods *C*, *C* pass practically air-tight. These rods are insulated from the smaller rods *H*, *H* which they contain, and which carry the carbons. *N* is the negative and *P* the positive

[1] *Electrical World*, **22**, 1893.

pole as commonly used. The latter is represented partly in section to show the cavity *S* in which the substance whose spectrum is desired is placed. The carbon *N* can be raised and lowered by means of the rack *R* and pinion *G*, and the carbon

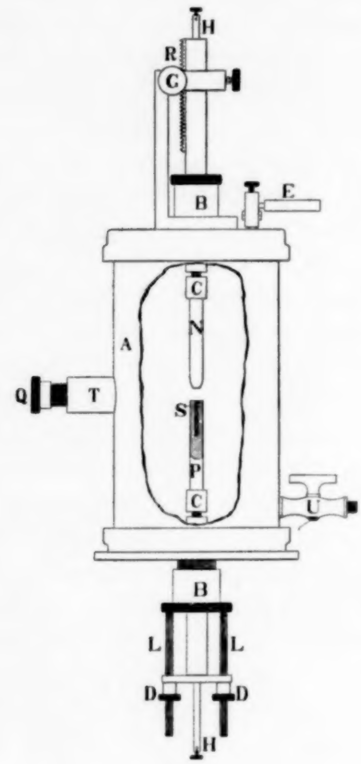


FIG. 1.

P can be brought to the proper position with the nuts and screws *D*, *D* and *L*, *L*. The light reaches the slit of the spectroscope by passing through the side tube *T*, which is closed at *Q* with a plane quartz disk. The object in using quartz instead of glass is of course to avoid, as far as possible, the absorption of the ultra-violet light. The air is pumped in through the tube *U*, which can be opened and closed by means of the stopcock *V*,

and the pressure is given by a Bourdon gage E, reading from one to twenty atmospheres.

METHOD OF PHOTOGRAPHING.

For the purpose of accurate comparison it was necessary to obtain side by side photographs of the spectra of the substance in question as given by the arc under the two pressures used, the lowest of which was always that of one atmosphere. It was also necessary to guard, as far as possible, against any accidental movement of the camera or other part of the apparatus during the exposure and to be able to surely detect any such accidental disturbance should it occur, since any slight movement of the apparatus during, and especially between successive exposures on the same plate, would necessarily lead to false results.

The first of these requirements, that is the obtaining of the spectra in such way that they could be accurately compared, was met in the same manner (and with the same apparatus) that Professor Rowland met a similar requirement in the comparison of solar and arc spectra, that is by providing the camera, which takes a nineteen by one and a quarter-inch plate, with a rotating shutter so constructed that in one position it shields the sides along the entire plate and leaves a narrow middle strip exposed, while in a certain other position it shields the middle strip and exposes the sides. In nearly every case the middle strip was exposed to the arc under pressure, after which the air was let out from the cylinder, the shutter adjusted and the sides of the plate exposed to the arc at atmospheric pressure.

The method used at first for detecting accidental disturbances was as follows: By means of an auxiliary shutter a small portion of the middle strip was exposed to the solar spectrum, all other parts of the plate being shielded, then the remainder of this strip to the arc under pressure, then the corresponding sides to the arc at atmospheric pressure, and finally the remaining portions to the solar spectrum. This process secured a short section of solar spectrum, the middle portion of which was exposed

before, and the sides after the exposures to the arc. Consequently any disturbance of the apparatus between the first and and last exposures was shown by breaks in the solar lines. It soon became evident, however, that this method, though accurate, was not necessary, since the lines of the carbon bands, some of which occur on nearly every plate, are never measurably displaced, and therefore serve perfectly to detect any disturbance of camera or other part of the apparatus during or between the exposures.

METHOD OF MEASURING.

The shifts of a few lines were determined by direct observations with a micrometer eyepiece, but in all other cases the measurements were carefully made on photographs with a most accurate dividing engine especially constructed by Professor Rowland for this sort of work, and used in determining Rowland's Table of Standard Wave-lengths. The dividing engine and the micrometer eyepiece are both constructed to read directly to hundredths of a millimeter, and may be estimated to thousandths of a millimeter.

Most of the plates were taken in the second spectrum, where the dispersion is a little more than one millimeter per Ångström unit, though a few were taken in the first, where the dispersion is one-half that of the second, and many in the third, where it is three halves that of the second. In all several hundred negatives were secured and the shifts determined of those lines whose positions were well defined by reason either of their sharpness or of their reversals.

To facilitate the measuring of the shifts and at the same time to increase the accuracy a system of double cross-hairs was placed, as shown in Fig. 2, in the field of the microscope. In the process of measuring the microscope was kept fixed and the negative moved along by the micrometer screw until the cross *a* was on the center of a given line in the middle strip, that is a line produced by the arc under pressure, when a reading was taken. The plate was then moved forward by the screw

until the crosses *bb* were on the center of the same line as formed on the sides of the plate by the arc at atmospheric pressure, when another reading was taken, and so on for other lines. The plate was then reversed and the same process repeated.

Let s be the shift of any line, and l the difference in readings that would be given by the crosses *a* and *bb* when there is no

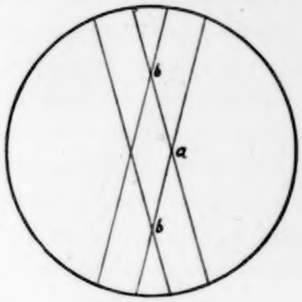


FIG. 2.

shift, and let the direct reading of the crosses *bb* be λ and the reversed d. By "direct reading" is meant that which is obtained when the plate is being moved so that the successive lines to come into the field of the microscope are of increasing wave-length, and by "reversed reading" that obtained when the plate is moving so that the successive lines seen through the microscope are of decreasing wave-length. The direct reading, therefore, given by *a* is $\lambda - l \pm s$, and the reversed $d - l \mp s$. Evidently $\lambda + d$ is a constant for all the lines on any one plate, from which, if the reversed readings be subtracted, remainders, "corrected reversed," will be obtained equal respectively to λ and $\lambda + l \pm s$. Consequently for any one line the average of the two readings "direct" and "corrected reversed" given by the crosses *bb* is λ, and that of those given by the cross *a*, $\lambda \pm s$, their difference being the shift $\pm s$. By this means the accuracy of the measurements is rendered quite considerable. In the case of extra good lines the error should not exceed from two to three thousandths of an Ångström unit.

EXPERIMENTAL RESULTS.

A little preliminary work was done with the arc spectra, at atmospheric pressure, of cadmium and a few other elements. The chief changes found were those of intensity and width of the lines, both of which increased with increase of material. A large amount of material in the arc caused the reversal of many lines, but in every case examined the reversal coincided very closely if not exactly with the position of the corresponding fine lines as produced by a small amount of the substance.

The element whose spectrum as formed under pressure was first examined was cadmium, and it was at once noticed that the positions of its lines were very appreciably changed by a pressure of even three or four atmospheres. Subsequently the lines of a large number of other substances were similarly examined, and in every case their positions, except those of lines of certain bands, were more or less changed.

That this change in position of the lines is not due to any strain or movement of some portion or other of the apparatus may be shown in several ways, but it is easiest and also best shown by the fact that on the same plate lines due to different substances are displaced to very different extents, whereas they should be equally displaced if the displacement were due to a disturbance of the apparatus while photographing. Nor is the observed shift of the lines due to unsymmetrical broadening, since in many cases equally fine and sharp lines were obtained at high and normal pressures, the only important difference between the lines as obtained under the two conditions being that of position. Not only were numerous lines of this character photographed, but the evidence as furnished by the negatives was also checked and confirmed by a number of eye observations, especially on the cadmium lines λ 6438.680 and λ 5086.001, and the sodium lines D_1 and D_2. By filling the positive pole with fused potassium sulphate, which usually, like the specimen used, contains more or less sodium, it was easy at any pressure, up to ten or more atmospheres, to get the sodium lines D_1 and D_2, and to retain

them some minutes as beautifully fine and sharply reversed lines, differing in no respect from the same lines as obtained at very different pressures except quite decidedly in position. A further reason for the statement that the shift is not due to unsymmetrical broadening is found in the fact that the wave-lengths of all fine and sharp lines, and also of the reversals of heavy ones, increase with increase of pressure around the arc, no matter how the lines may spread out, symmetrically or chiefly towards either side. A splendid example of this is furnished by the pair of sodium lines λ 3302.504 and λ 3303.119. These lines are quite unsymmetrical — spreading chiefly towards the violet or more refrangible end of the spectrum — but their reversals are greatly shifted in the opposite direction. Neither is the shift due to the disappearance of one line and the appearance of another of slightly different wave-length, because the wave-length increases regularly and not by jumps as the pressure is increased; and, besides, it is not difficult to observe, while the pressure is being slowly let off, either a fine line or the reversal of a heavy one gradually change in position without alteration in width or any other respect.

It has been suggested by Schuster[1] that this shift of spectral lines is possibly due to the "proximity of molecules vibrating in equal periods." If this supposition is correct, then, of course, the shifts of the lines should be greater at any given pressure, as Schuster says, the greater the amount of material used that produces them. However, many experiments both before and since the appearance of Schuster's paper show that this is not the case. Among the substances that have been most fully tested in this respect are iron, titanium, copper, and zinc. The carbons used, though reasonably pure, contained a considerable number of impurities in sufficient amounts to give some of their strongest lines, and among these substances were iron, titanium, and copper, each of which gave some very fine but quite measurable lines. The amounts of these substances were then gradually increased until they were as great as possible. In the case of

[1] *Ap. J.*, April 1896.

iron, copper, and zinc, solid rods of the metals were finally used, but in every instance the shift of a given line of any substance remained constant for any definite pressure, showing that it depends upon the absolute pressure and not upon the partial pressure of the gas or vapor producing the line in question.

More recently it has been suggested by Fitzgerald[1] that a "*vera causa* for some shift towards the red in molecules causing light" is the increase of the specific inductive capacity, due to increase of density, of the gas surrounding the arc. This suggestion is based, of course, upon the assumption, possibly a correct one, that "electric forces are at least a part of the forces affecting the periods of vibration." The correctness of this suggestion has not been submitted to actual experimental tests, nor does it seem very easy to do so, at least not directly, since the differences in the specific inductive capacities of gases are not sufficient to produce changes in the shifts greater than the errors of observation, even if the shifts are due entirely to the cause suggested. No matter what theory or suggestion is advanced, it must be remembered that it is imperfect if it does not account in some way for the important fact that at least many elements produce two or more groups of lines, differing greatly from each other in the magnitude of their shifts.

If, as many believe, the temperature of the electric arc is that of boiling carbon it would seem natural to suppose that it would rise with increase of pressure. Very little seems to have been done to test this point, but a number of experiments as conducted by Wilson,[2] and later by Wilson and Fitzgerald[3] have given conflicting results. However, whether pressure causes an increase or a decrease of temperature, in either case the shifts of the spectral lines may conceivably be due to a change in temperature rather than pressure, and experiments were undertaken to clear up this point. In accordance with Wilson and Gray's[4] work, which indicates that the temperature of the negative pole is much less than that of the positive, a long arc, due to a fairly

[1] *Ap. J.*, March 1897.

[2] *Proc. R. Soc.*, May 30, 1895.

[3] *Ap. J.*, February, 1897.

[4] *Proc. R. Soc.*, November 24, 1894.

heavy current, was formed at right angles to the slit of the spectroscope, and one part of a photographic plate exposed to the spectrum due to the arc close to the positive and the other part to the spectrum as formed by the arc near the negative pole. No change, however, was detected in the position of the lines. Another method of testing the same point was to vary between wide limits the strength of current used, since the temperature, according to Moissan,[1] probably rises with increase of current. The extreme currents used were two amperes and 180 amperes respectively, but the positions of the lines appeared to remain absolutely unchanged. Further tests were made on the sodium line D_2 (one of the most sensitive of all lines examined) as produced first in the Bunsen flame and then in the electric arc; but while the temperatures of the vapor in the two cases were probably widely different the position of the line remained the same, as nearly as could be determined. Again one would certainly expect the outer envelope of an electric arc to be much cooler than the core, but the position of a reversal given by the former is exactly the same as that of a fine sharp line produced by the latter. The expression, "temperature of the arc or flame," is used with hesitation, since so little is known of the mean condition of the molecules producing light, but the negative results of all the above experiments give every assurance that the shifts of the lines are not ordinary temperature effects. It should be stated, however, that the luminous intensity of the arc greatly increased, especially where metallic poles were used, with increase of pressure.

The numerous negatives obtained, as well as the eye observations made, show that the general effect of pressure is to broaden the lines and to bring out their reversals. However, this is not always the case, since lines often appear quite as fine and sharp at one pressure as at another; and in all probability the broadening of the lines is due, chiefly at least, to an increase of density of the gas producing them, since it is always greater the greater the amount of the substance used in the arc. This

[1] *Annals de Chimie et de Physique*, October 1896, p. 231.

idea is also in accord with Schuster's[1] observation that when gases are mixed in different proportions the lines of any one become sharper when it is present in smaller quantity, though the total pressure may remain the same.

The lines of the cyanogen bands came out more strongly on my plates under pressure (the pressure being due to atmospheric air), but never showed much if any shift, which fact furnished conclusive evidence that the shifts of other lines were due to real changes in wave-frequencies and not to some disturbance of the apparatus, since they were all photographed simultaneously on the same plate, the lines of the cyanogen bands never being appreciably displaced while other lines were.

The shift or displacement of any line is directly proportional to the excess of pressure above one atmosphere (the position of the line as formed at atmospheric pressure being taken as its normal or zero position) and is always towards the less refrangible or red end of the spectrum. The same law has been shown by Mohler[2] to hold for pressures below one atmosphere—the shift in this case being to the violet. The shift is very different for the lines of different elements, and also, in many cases at least, for different groups of lines of the same element. In particular the shifts of the several series of lines (as given by the alkalies), principal and subordinate, are by no means equal, even when the lines are of approximately the same wave-length, as shown by the tabulated results in Table I. Lines of the second subordinate series seem to shift about twice as much as those of the first, which in turn are displaced to an extent approximately twice that of the lines of the principal series. A few iron lines, each of which is more hazy or softer than the average line of this element, are shifted about three times as much as other lines of the same substance. Again, all the nebulous or hazy copper lines examined shift to approximately the same extent (allowance being made for wave-length) but much more than do other lines of the same element. It is worth noting that these nebulous lines of copper were best obtained when both poles of the

[1] *Ency. Brit.*, "Spectroscopy."

[2] *Ap. J.*, October 1896.

arc were metallic rods, brass and copper or both copper, and that apparently the only effect of pressure on them was to greatly increase their wave-lengths. In this connection mention must be made of the calcium line *g*, which is shifted about twice as much as the similar calcium lines H and K. The same thing is also true of the three corresponding lines of strontium and of barium.

Similar lines of any given element, that is, lines belonging to the same series, or to no series but of the same character, shift to extents proportional to their wave-lengths. The most conclusive evidence of this proportionality was furnished by lines of different orders of spectra that appeared on the same plate. Thus ultra-violet lines of the third order were often found on the same plate with similar lines of the second of longer wave-length but due to the same element, and their measured shifts were approximately the same. Since the wave-length of a line of the third order is to that of one of the second that occurs at the same place as two to three, while the dispersion in the third order is to that in the second as three to two, it follows that constancy of measured shifts means that it is proportional to wave-length. For the sake, therefore, of comparison it seemed advisable to reduce the shifts of all lines to what they would be at some definite wave-length; the one chosen being 4000 Ångström units, since most of the work was done in that neighborhood.

It should be stated that in some cases the values obtained for the shifts of the lines may have been due in a measure to unsymmetrical broadening; but this has certainly not led to much error, since as already stated, only those lines were used which could be fairly accurately measured, that is, those which were either comparatively narrow or else reversed.

DESCRIPTION OF TABLE I.

The results of the numerous measurements are given in Table I, in which the uppper numbers in the line of each wave-length are the observed shifts in thousandths of an Ångström unit, and the lower their values reduced to wave-length 4000. In many cases the observed shift, as given, is the average of fairly con-

cordant measurements of the same line on different plates. The different pressures used are given in atmospheres at the heads of the columns, and are greater by unity (since the lower pressure of each experiment was always one atmosphere) than the difference in pressure, $p-p_0$, to which the shift was actually due.

In most cases the wave-lengths are taken either from Professor Rowland's Table of Solar Spectrum Wave-lengths, in process of publication in the ASTROPHYSICAL JOURNAL, or from a former table of his published in *Astronomy and Astro-Physics*.[1] Some are taken from the papers of Kayser and Runge, and a few from other sources, but it was found necessary, for the want of suitable tables, to determine a number of them by comparison with known lines in their neighborhood, and since exact wave-lengths are not essential to this work, only such approximations of them are given as will serve to surely identify the lines in question.

TABLE I.

Showing the pressure in atmospheres and the observed resulting shifts ($\Delta\lambda$) in thousandths of an Ångström unit, and the same reduced to wave length 4000.

ALUMINIUM.

Wave-length λ	$\Delta\lambda$ observed and Pressure in atmospheres, followed by $\Delta\lambda$ reduced to λ 4000										
	4¼	4¾	6	7	7½	9	10½	10¾	11¾	12¾	14
3082.27 ..				38 50		38 50					
3092.84 ..				40 52		38 50					
3944.114..		17 17	29 29					44 45	49 50	48 49	68 69
3961.674..	17 17	25 25	26 26	37 37	36 36		44 45	40 41	57 58	52 53	76 77
Average.	17 17	21 21	28 28	38 46	36 36	38 50	44 45	42 43	53 54	50 51	72 73

NOTE.—Several lines of the aluminium oxide band, 4842–5041, were measured on two plates which were taken at different pressures. The shift, if anything, was very small.

[1] *A. and A.*, **12**, 1893.

TABLE I.—*Continued.*

ANTIMONY.

Wave-length λ	Δλ observed and Pressure in atmospheres, followed by Δλ reduced to λ 4000						
				6¾			
3267.6				21 / 26			

BARIUM.

Wave-length λ	Δλ observed and Pressure in atmospheres, followed by Δλ reduced to λ 4000						
	7	8	8½	10	10½	11	12
4432.13	45 / 41						78 / 71
4506.11	36 / 32					56 / 50	69 / 61
5535.69	32 / 23	52 / 38		56 / 40	68 / 50		80 / 58
Group A.							
Average	38 / 32	52 / 38		56 / 40	68 / 50	56 / 50	76 / 63
4554.211	23 / 20		24 / 21	32 / 28	34 / 30	40 / 35	
4934.237			23 / 19	34 / 28			
Group B.							
Average	23 / 20		24 / 20	33 / 28	34 / 30	40 / 35	
3910.04							117 / 120
3935.87							119 / 121
3993.60							126 / 126
4726.63							135 / 115
Group C.							
Average							124 / 120

TABLE I.—*Continued.*

ARSENIC.

Wave-length λ	Pressure in atmospheres, followed by Δλ observed and Δλ reduced to λ 4000						
			8½	9	10		
2745.09					29 20		
2780.30					32 22		
2860.54					28 20		
2898.83			28 20	25 18			
Average			28 20	25 18	30 21		

BERYLLIUM (GLUCINUM).

Wave-length λ	Pressure in atmospheres, followed by Δλ observed and Δλ reduced to λ 4000						
				7½			
3130.6				14 11			
3321.3				24 19			
3321.5				24 19			
Average				21 16			

BORON.

Wave-length λ	Pressure in atmospheres, followed by Δλ observed and Δλ reduced to λ 4000						
		8	8½	9	9¼		
2496.867		30 19	37 23	37 23	40 25		
2497.821		30 18	30 18	35 22			
Average		30 19	34 21	36 23	40 25		

TABLE I.—*Continued.*

BISMUTH.

Wave-length λ	Pressure in atmospheres, followed by Δλ observed and Δλ reduced to λ 4000					
			10	13½		
2898.08			26 35			
2989.15			34 45			
3397.31				48 57		
Average			30 40	48 57		

CÆSIUM.

Wave-length λ	Pressure in atmospheres, followed by Δλ observed and Δλ reduced to λ 4000					
			6	7		
4555.44			138 121	123 108		
4593.34			95 82	78 68		
Average			117 102	101 88		

CARBON.

Wave-length λ	Pressure in atmospheres, followed by Δλ observed and Δλ reduced to λ 4000					
		8	8½	9	9¼	10	
2478.661		24 38	20 32	24 38	23 38	26 42	

NOTE.—A number of "cyanogen" lines were measured at various pressures, and found to shift very little, if at all.

TABLE I.—*Continued.*

CADMIUM.

Wave-length λ	Pressure in atmospheres, followed by Δλ observed and Δλ reduced to λ 4000						
	7	7½	8	9	9½	10	10½
3261.17					22 18		
3466.33					23 20		28 25
3610.66						22 20	
No series.							
Average					23 19	22 20	28 25
6438.680	Reduced to λ 4000 and for a pressure of 12 atmospheres Δλ=80.[1]						
3403.74					34 29		30 25
3467.76					31 27	21 18	23 20
3613.04					18 16	21 19	
First subordinate series.							
Average					28 24	21 19	27 23
3081.03	53 40			80 60			
3133.29	60 48			65 51			
3252.63		38 31			57 47	69 56	
4678.347				47 54	68 79		
4800.080			36 43	40 48	58 70		
5086.078	42 53	47 59	50 63	54 67	56 70		66 82
Second subordinate series.							
Average	52 47	43 45	43 53	57 56	60 67	69 56	66 82

[1] Mean of several concordant eye observations by different persons and at different pressures.

Wave-length λ	Δλ observed and Pressure in atmospheres, followed by Δλ reduced to λ 4000											
	3	4½	5	6	7	8	9	10	10½	11	12½	14½
K 3933.825									25 / 25	24 / 24	28 / 28	
H 3968.625									22 / 22	26 / 26	33 / 33	
4283.169			12 / 13	17 / 18		27 / 29				29 / 31	28 / 30	
4289.525			15 / 16	16 / 17		19 / 20				33 / 35	38 / 40	
4299.149		8 / 8	16 / 17	12 / 13		20 / 22				33 / 35	32 / 34	
4302.692				11 / 12		25 / 27				28 / 31	27 / 30	
4307.91				25 / 27						27 / 30		
4318.80						25 / 27		22 / 24		27 / 30		
Group A Average		8 / 8	14 / 15	16 / 17		23 / 25		22 / 24	24 / 24	28 / 30	31 / 33	
3158.98						53 / 42	60 / 47					
3179.45						47 / 37						
g 4226.904						40 / 42		45 / 48		48 / 51	53 / 56	82 / 87
4435.13[1]										74 / 81		
4454.97[1]										73 / 80		
5588.985					47 / 66		49 / 68			50 / 70		
5594.691					47 / 66		57 / 80			52 / 72		
5598.711					49 / 68		55 / 74			47 / 63		
5603.083						40 / 56						
Group B Average					48 / 67	45 / 44	55 / 67	45 / 48		57 / 69	53 / 56	82 / 87
4425.61										76 / 84		
4435.86										80 / 88		
4456.08										71 / 79		
1st subordinate ser. Average										76 / 84		
6102.99	44 / [2]67		77 / [2]117			109 / [2]166	133 / [2]203					
6122.46	35 / [2]53		53 / [2]82			91 / [2]140			141 / [2]215			
6162.46		55 / [2]84					96 / [2]148		139 / [2]214			
2d subordinate ser. Average	40 / 60	55 / 84	65 / 100			100 / 153	115 / 176		140 / 215			

[1] Occurs with lines of 1st subordinate series, and shifts to the same extent. [2] Eye observations.

CERIUM.

Wave-length λ	Pressure in atmospheres, followed by Δλ observed and Δλ reduced to λ 4000						
			6½	8			
3895.224			21 20				
3896.917			14 14				
3898.4			23 22				
3912.5			17 17				
3917.7			13 13				
3918.4			8 8				
3919.9			20 20				
3921.6			22 22				
3929.3			17 17				
3931.2			17 17				
3940.4			19 19				
3941.1			8 8				
3949.2				8 8			
3953.7			8 8				
3955.4			10 10				
3957.4				15 15			
3961.0			7 7				
3964.6			6 6				
3971.8			12 12				
3972.2			17 17				
3975.1			19 19				
3978.7			7 7	9 9			
3984.7			12 12				
3989.5			11 11				
3992.5			15 15	21 21			
3993.0			17 17				
Average			14 14	13 13			

TABLE I.—*Continued.*

CHROMIUM.

Wave-length λ	Pressure in atmospheres, followed by Δλ reduced to λ 4000 / Δλ observed and											
	4¾	6	7	9¼	10	10½	11	11½	12¼	12½	14	14½
3886.932..	4 / 4						27 / 26				37 / 36	
3919.309..		16 / 16					20 / 20			12 / 12	30 / 29	
3941.637..	7 / 7						34 / 33				33 / 32	
3963.831..	7 / 7	12 / 12	16 / 16	25 / 25		23 / 23	27 / 27			36 / 36	47 / 47	
3976.839..	14 / 14	12 / 12	12 / 12				28 / 28				38 / 38	
3984.059..	12 / 12						34 / 34				49 / 49	
4026.318..			19 / 19									
4254.505..	3 / 3	9 / 9	18 / 19				24 / 26				24 / 26	28 / 30
4266.894..					16 / 17							
4274.958..	13 / 14	10 / 11					29 / 31		22 / 24		38 / 40	29 / 31
4280.556..							28 / 30	22 / 23				
4289.885..							32 / 34				44 / 41	
Average	9 / 9	13 / 14	15 / 16	25 / 25	16 / 17	23 / 23	28 / 29	22 / 23	22 / 24	24 / 24	37 / 38	29 / 31

COLUMBIUM (NIOBIUM).

Wave-length λ	Pressure in atmospheres, followed by Δλ reduced to λ 4000 / Δλ observed and						
			8½	9½			
3914.8				28 / 27			
3937.7				13 / 13			
4059.0			22 / 22	24 / 24			
4079.9			29 / 30	31 / 32			
Avearge			26 / 26	24 / 24			

TABLE I.—*Continued.*

COPPER.

Wave-length λ	Pressure in atmospheres, followed by $\Delta\lambda$ observed and $\Delta\lambda$ reduced to λ 4000						
		7	8	12¼	12½	13	13½
2883.03		8 11	7 10				
3010.92		11 15					
3036.17		9 12					
3073.89		7 10					
3094.07		13 17					
3247.680				25 30	28 33		28 33
3274.092				20 25	32 38	30 36	36 41
3317.28			13 16				
3337.095		11 13	11 13				
3476.07		8 10	14 16				
3483.82		12 14	17 19				
3520.07		16 18					
3524.31		12 14	19 21				
3533.84		13 15					
3545.05		17 19					
3599.20		15 17	17 19				
3621.33		10 11					
3636.01		16 18					
3684.75		13 15					
5105.75			20 16				
Lines of small shift. Several others of this set were measured							
Average		12 14	15 16	23 28	30 36	30 36	32 37
3365.46		27 32	28 33				
3381.52		24 28					
3620 47		36 40					

TABLE I.—*Continued.*

COPPER—*Continued.*

Wave-length λ	Pressure in atmospheres, followed by $\Delta\lambda$ observed and $\Delta\lambda$ reduced to λ 4000						
		7	8	12¼	12½	13	13½
3741.32		38 35					
3860.64		37 35					
5218.45			33 43				
Lines of medium shift							
Average		35 31	33 36				
4177.87		83 87					
4249.21		54 57					
4275.32		60 64					
4378.40		74 81					
4415.79		73 80					
4587.19		64 74					
5292.75			52 68				
Lines of large shift. Several others belong to this set. Most of these lines are "soft" and broad							
Average		68 74	52 68				
5153.33			21 27				
5220.25			23 30				
First subordinate series							
Average			22 29				
4480.58		44 50					
4531.04		46 53					
Second subordinate series							
Average		45 52					

TABLE I.—*Continued.*

COBALT.

Wave-length λ	Pressure in atmospheres, followed by $\Delta\lambda$ reduced to λ 4000 and $\Delta\lambda$ observed						
		$9\frac{3}{4}$	$11\frac{1}{4}$	$12\frac{1}{2}$	$14\frac{1}{2}$		
3354.515				19 23	33 39		
3361.413				18 22			
3395.016				20 24	22 26		
3405.255		17 20					
3409.336		14 16			23 27		
3417.384				23 27	29 34		
3461.326		18 21					
4121.476			20 19				
Average		16 19	20 19	20 24	27 32		

ERBIUM.

Wave-length λ	Pressure in atmospheres, followed by $\Delta\lambda$ reduced to λ 4000 and $\Delta\lambda$ observed						
				8			
3988				30 30			

GERMANIUM.

Wave-length λ	Pressure in atmospheres, followed by $\Delta\lambda$ reduced to λ 4000 and $\Delta\lambda$ observed						
			7	$8\frac{1}{2}$	9		
3039.198				22 29	24 31		
3269.628				24 29	22 27		
4226.724			30 28				
Average			30 28	23 29	23 29		

TABLE I.—*Continued.*

GOLD.

Wave-length λ	Pressure in atmospheres, followed by Δλ observed and Δλ reduced to λ 4000						
			7	10	10½		
3122.88					20 / 26		
3898.04					40 / 41		
3909.54					25 / 26		
4041.07			20 / 20	25 / 25			
4065.22			34 / 33	34 / 33			
Average			27 / 27	30 / 29	26 / 29		

INDIUM.

Wave-length λ	Pressure in atmospheres, followed by Δλ observed and Δλ reduced to λ 4000						
	7	9½	10	10½	11½	12½	14½
3256.17 No series		19 / 23	23 / 28				
3258.66 1st subordinate series		36 / 44	37 / 45				
2932.71	43 / 60						
4102.00					69 / 68		
4511.345 2d subordinate series		73 / 65	83 / 74	81 / 72		102 / 90	125 / 111
Average	43 / 60	73 / 65	83 / 74	81 / 72	69 / 68	102 / 90	125 / 111

IRON.

Wave-length λ	Δλ observed and Pressure in atmospheres, followed by Δλ reduced to λ 4000													
	4¾	6	7	9	10	11	11¼	11½	12¼	12½	12¾	13	14	14½
3997.547 . . .							29 / 29							
4005.408 . . .			15 / 15		23 / 23	32 / 32								
4009.864 . . .					18 / 18									
4045.975 . . .	8 / 8	9 / 9					20 / 20							
4063.759 . . .							23 / 23							
4181.919 . . .					25 / 26									
4199.267 . . .					24 / 25									
4207.291 . . .					17 / 18									
4219.516 . . .					17 / 18									
4236.112 . . .	8 / 8		16 / 17			25 / 26								
4250.945 . . .			11 / 11		25 / 26	29 / 31				33 / 35	24 / 25		32 / 34	
4271.934 . . .	8 / 8	12 / 13	13 / 14			23 / 25	26 / 28	19 / 20	26 / 28	22 / 23	24 / 26		28 / 30	28 / 30
4294.301 . . .					23 / 25									
4298.195 . . .					24 / 26									
4325.939 . . .							24 / 26				30 / 32	32 / 35	31 / 34	
4377.948 . . .								19 / 21						
4382.928 .					22 / 24				24 / 26		25 / 27	32 / 35		28 / 31
4383.720 . . .								26 / 28						
4404.927 . . . Many other lines belong to this group								21 / 23						
Average	8 / 8	11 / 11	14 / 14		22 / 23	27 / 29	24 / 25	21 / 23	25 / 27	28 / 29	26 / 28	32 / 35	30 / 32	28 / 31
4222.382 . . .									66 / 70					
4227.606 . . .									73 / 77					
4233.772 . .									75 / 79					
4236.112 . . .			25 / 26	56 / 59	57 / 60				75 / 79					
4250.287 . . .			54 / 57	90 / 95	80 / 84	77 / 81			74 / 78			81 / 86		
4260.647 . . .			45 / 47	68 / 72	63 / 66	65 / 69			70 / 74			81 / 86		
4271.934 . . .			56 / 59	92 / 98	70 / 74	77 / 82			79 / 84					
Average . .			45 / 47	76 / 81	68 / 71	73 / 77			73 / 77			81 / 86		

NOTE.—Other lines, among them 5569.77, 5573.05, and 5586.92, belong to this group.

TABLE I.—*Continued.*

LANTHANUM.

Wave-length λ	Pressure in atmospheres, followed by Δλ observed and Δλ reduced to λ 4000						
			8	9			
3921.695				21 21			
3929.363			13 13	32 33			
3949.199			26 26	24 24			
3995.899			21 21	35 35			
4031.865			7 7	14 14			
4043.054			29 29				
4077.498			17 17				
4086.861			21 21				
Average			19 19	25 25			

LEAD.

Wave-length λ	Pressure in atmospheres, followed by Δλ observed and Δλ reduced to λ 4000						
		9	11	11¼	13½		
3639.728					63 70		
3683.622					70 76		
4058.041		49 48	55 54	49 48			
Average		49 48	55 54	49 48	67 73		

LITHIUM.

Wave-length λ	Pressure in atmospheres, followed by Δλ observed and Δλ reduced to λ 4000													
	2½	3	4	4½	5	5½	6	6½	7	8	9	9½	10	10½
6708.2	11 / [1]18	14 / [1]24	18 / [1]30				40 / [1]66		31 / [1]52					78 / [1]130
3232.77 .... Principal series										66 / 53	74			
6103.77 .... 1st subordinate series	25 / [1]38	24 / [1]37		37 / [1]56		50 / [1]77		76 / [1]116				116 / [1]177		
4972.11..... 2d subordinate series										181 / 222				

[1] Eye observation.

MANGANESE.

Wave-length λ	Pressure in atmospheres, followed by Δλ observed and Δλ reduced to λ 4000									
	4¾	6	7	10½	11	11¼	11½	12½	12¾	14
4018.269...		13 / 13	12 / 12					32 / 32	27 / 27	37 / 37
4026.583...	8 / 8				17 / 17					
4030.947...						19 / 19			33 / 33	34 / 34
4035.883...	8 / 8	18 / 18	13 / 13						26 / 26	32 / 32
4061.881...						20 / 20				
4235.298...	10 / 10		21 / 22							
4235.450...	8 / 8		17 / 18	25 / 27	34 / 37				29 / 32	33 / 36
4239.890...	12 / 13	20 / 21					29 / 31	45 / 48		44 / 47
4257.815...	13 / 14	20 / 21	21 / 22	23 / 25	36 / 39				43 / 47	43 / 47
4266.081...	13 / 14	19 / 20	21 / 22	35 / 38			37 / 40	38 / 41	37 / 40	43 / 46
4281.257...	9 / 10		20 / 21	34 / 37	40 / 43		36 / 39	38 / 41	35 / 38	41 / 44
4284.223...	11 / 12									
Average .	10 / 11	18 / 19	18 / 19	30 / 32	32 / 34	20 / 20	34 / 37	38 / 40	33 / 35	38 / 40

MAGNESIUM.

Wave-length λ	Pressure in atmospheres, followed by Δλ observed and Δλ reduced to λ 4000						
	7½	8	8½	9	10	11	13
2795.632			8 12	19 27			
2802.805			18 26	16 23			
[1]2852.239 No series	12 17	6 9			23 32	21 30	29 37
Average	12 17	6 9	13 19	18 25	23 32	21 30	29 37
3829.501				33 34	28 29		
3832.450				30 31	31 32		
3838.435 1st subordinate series				40 41	30 31		
Average				34 35	30 31		
5167.497				66 51			
5172.856				62 48			
5183.791 2d subordinate series				47 36			
Average				58 45			

[1] This line, much the strongest in the spectrum of magnesium, is nearly coincident with, and consequently almost always obscures a much weaker line of the first subordinate series.

MERCURY.

Wave-length λ	Pressure in atmospheres, followed by Δλ observed and Δλ reduced to λ 4000						
			10	11			
3650.3			63 70	63 70			
5461.0			90 66				
Average			77 68	63 70			

TABLE I.—*Continued.*

MOLYBDENUM.

Wave-length λ	Pressure in atmospheres, followed by Δλ observed and Δλ reduced to λ 4000						
			9	11½			
3132.749				40 31			
3158.3				34 27			
3170.5			33 28	40 32			
3194.2			22 18	41 33			
Average			28 23	39 31			

NICKEL.

Wave-length λ	Pressure in atmospheres, followed by Δλ observed and Δλ reduced to λ 4000						
			9¾	12½	14½		
3391.180				17 14			
3413.637				23 19			
3414.092				23 19			
3437.447			23 20		39 34		
3458.606				31 27	33 29		
3461.322			18 16	25 23			
3500.993				27 24	40 35		
3515.207				38 34	45 41		
3524.677				35 30			
5155.937			18 24				
Average			20 20	26 24	39 35		

TABLE I.—*Continued.*

NEODYMIUM.

Wave-length λ	Pressure in atmospheres, followed by Δλ observed and Δλ reduced to λ 4000						
				9			
4279.874				18 17			
4281.0				7 7			
4284.8				6 6			
4302.7				7 7			
4319.1				12 11			
4334.3				5 5			
4348.0				11 10			
4362.2				5 5			
4385.8				15 14			
4401.0				14 13			
4420.7				6 5			
And many others							
Average				10 9			

OSMIUM.

Wave-length λ	Pressure in atmospheres, followed by Δλ observed and Δλ reduced to λ 4000						
			12¾	13			
4260.993				17 16			
4520.633			20 18	18 16			
Average			20 18	18 16			

PALLADIUM.

Wave-length λ	Pressure in atmospheres, followed by Δλ observed and Δλ reduced to λ 4000						
			12	13½			
3373.139				39 / 33			
3404.725				47 / 40			
3421.367			20 / 17	44 / 38			
3433.578				36 / 31			
3441.539				36 / 31			
3460.884			21 / 18	31 / 27			
3481.300			24 / 21	29 / 25			
3489.915			22 / 19	34 / 30			
3609.696			24 / 22	48 / 44			
3634.841				45 / 41			
3690.483			19 / 17	31 / 28			
Average			22 / 19	38 / 33			

PLATINUM.

Wave-length λ	Pressure in atmospheres, followed by Δλ observed and Δλ reduced to λ 4000						
		12¼	12¾	13	14¼		
2830.41				22 / 16			
2893.98				20 / 14			
2897.99				17 / 12			
2929.91			16 / 12	24 / 17			
2998.079		20 / 15	23 / 17	25 / 18	18 / 13		
3042.745				25 / 18	31 / 23		
3064.824					21 / 16		
4442.723		23 / 25		28 / 31			
Average		21 / 20	20 / 15	23 / 18	24 / 18		

TABLE I.—*Continued.*

POTASSIUM.

Wave-length λ	Pressure in atmospheres, followed by Δλ observed and Δλ reduced to λ 4000						
			8	9			
4044.294			75 76	92 93			
4047.338			87 88	105 106			
Principal series							
Average			81 82	98 99			

RHODIUM.

Wave-length λ	Pressure in atmospheres, followed by Δλ observed and Δλ reduced to λ 4000						
		12	12¼	12¾	13	14½	
3399.839		20 17					
3412.417		27 23					
3435.039		19 16					
3462.184		21 19					
3474.920		23 20					
3479.053		25 22					
3502.674		24 21					
3507.466		29 26					
3626.744		18 16					
3658.135		17 15					
3666.366		25 23					
3690.853		29 27					
4211.304					43 45		
4374.981			28 31	34 38	41 45	34 37	
Average		23 20	28 31	34 38	42 45	34 37	

TABLE I.—*Continued.*

RUBIDIUM.

Wave-length λ	Pressure in atmospheres, followed by $\Delta\lambda$ observed and $\Delta\lambda$ reduced to λ 4000						
			8	8½			
4201.98			70 73	117 123			
4215.72			71 75	84 88			
Average			71 74	101 106			

RUTHENIUM.

Wave-length λ	Pressure in atmospheres, followed by $\Delta\lambda$ observed and $\Delta\lambda$ reduced to λ 4000						
				12			
3429.689				34 29			
3499.095				30 26			
3593.178				23 21			
3599.914				30 27			
3625.339				36 32			
3635.084				22 20			
3637.612				28 25			
3661.525				36 33			
3663.520				19 17			
3669.688				25 23			
3678.456				22 20			
3727.073				27 25			
3728.173				23 21			
3730.577				36 33			
Average				28 25			

TABLE I.—*Continued.*

SCANDIUM.

Wave-length λ	Pressure in atmospheres, followed by Δλ observed and Δλ reduced to λ 4000						
				12			
4247.0				22 21			
4314.3				30 28			
4320.9				26 24			
Average				26 24			

SILICON.

Wave-length λ	Pressure in atmospheres, followed by Δλ observed and Δλ reduced to λ 4000						
	8½	9	9½	10	11	11½	12
2506.994	13 21						
2516.210	12 19						
2519.297	18 28						
2524.206	13 21						
2528.599	22 34						
2881.695	20 28	21 29		25 35	31 40		
3905.660			31 32		44 45	40 41	39 40
Average	16 25	21 29	31 32	25 35	38 43	40 41	39 40

TABLE I.—*Continued.*

SILVER.

Wave-length λ	Pressure in atmospheres, followed by $\Delta\lambda$ observed and $\Delta\lambda$ reduced to λ 4000						
		8	9¾	12½	13		
3280.80		29 34	28 33	32 39			
3383.00			34 40	27 32	32 38		
Average		29 34	31 37	30 36	32 38		

SODIUM.

Wave-length λ	Pressure in atmospheres, followed by $\Delta\lambda$ observed and $\Delta\lambda$ reduced to λ 4000								
	3	3½	6½	7	7½	8	8½	9	10½
3302.504						47 57	67 81		
3303.119						61 74	57 70		
D_2 5890.182	[1] 30 20		[1] 69 47	62 42				94 62	[1] 121 82
D_1 5896.154		[1] 25 17	[1] 63 43	78 53				122 83	[1] 116 79
Principal series Average	30 20	25 17	66 45	70 48		54 66	62 76	108 73	119 81
5682.861					314 221	[1] 400 280		426 300	
5688.434	[1] 100 70	[1] 130 91			368 259	[1] 345 242		460 323	
2d subordinate series Average	100 70	130 91			341 240	373 261		443 312	

[1] Eye observation.

TABLE I.—*Continued.*

STRONTIUM.

Wave-length λ	Pressure in atmospheres, followed by Δλ observed and Δλ reduced to λ 4000						
	8½	10	10½	11	11½	12	
4077.885				26 26	38 39		
4215.703	27 28		32 34	32 34	43 45		
4742.07				36 42	31 37		
4784.43				35 42	29 35		
4812.01				19 23		24 29	
4832.23				41 50	40 48		
4876.35				33 40	33 40		
5222.43			35 46				
5225.35			46 60				
5229.52			34 44				
5238.76			34 45				
5257.12			46 60				
Group A							
Average	27 28		38 48	32 37	36 41	24 29	
3351.35			63 53				
3380.89			67 57			83 71	
3464.58						89 78	
4607.510	40 46	50 57		46 53		81 93	
4962.45				72 90		66 83	
Group B							
Average	40 46	50 57	65 55	59 72		80 81	

TABLE I.—*Continued.*

TANTALUM.

Wave-length λ	Pressure in atmospheres, followed by Δλ observed and Δλ reduced to λ 4000						
				12			
3918.6				13 13			
3922.9				19 19			
3931.1				11 11			
3970.3				16 16			
3982.1				18 18			
3988.9				23 23			
4003.9				14 14			
4007.0				14 14			
4027.1				18 18			
4030.1				21 21			
4061.6				15 15			
4064.8				20 20			
4105.2				18 18			
Average				17 17			

THALLIUM.

Wave-length λ	Pressure in atmospheres, followed by Δλ observed and Δλ reduced to λ 4000						
			9½	11			
3519.342			56 49	99 [1]87			
3529.58				86 [1]75			
Average			56 49	93 81			

[1] Good lines.

TABLE I.—*Continued.*

TUNGSTEN.

Wave-length λ	Pressure in atmospheres, followed by $\Delta\lambda$ observed and $\Delta\lambda$ reduced to λ 4000						
			$9\frac{1}{8}$	11			
4009.0			20 / 20	13 / 13			
4074.7				15 / 15			
Average			20 / 20	14 / 14			

TIN.

Wave-length λ	Pressure in atmospheres, followed by $\Delta\lambda$ observed and $\Delta\lambda$ reduced to λ 4000						
		$9\frac{3}{4}$	10	$12\frac{1}{2}$	13	$14\frac{1}{8}$	
2706.61			20 / 30				
2812.70			40 / 57				
2840.06			24 / 34				
2850.72			24 / 34				
2863.41			25 / 35				
3009.24			31 / 41				
3032.88			22 / 30				
3034.21			36 / 48				
3175.12				48 / 59		54 / 68	
3262.44		37 / 46		47 / 58	44 / 55	51 / 63	
3330.71				44 / 55		64 / 77	
Average		37 / 46	28 / 39	48 / 58	44 / 55	58 / 69	

TITANIUM.

Wave-length λ	8 (Δλ obs.)	8 (Δλ red.)	8½ (Δλ obs.)	8½ (Δλ red.)	9 (Δλ obs.)	9 (Δλ red.)	10½ (Δλ obs.)	10½ (Δλ red.)	11 (Δλ obs.)	11 (Δλ red.)	11½ (Δλ obs.)	11½ (Δλ red.)
	Pressure in atmospheres, followed by Δλ observed and Δλ reduced to λ 4000											
3186.564	25	20										
3192.120	27	22	26	21								
3200.034	22	18	31	25								
3222.970	19	15										
3234.635	15	12									26	21
3236.703	16	13									22	18
3239.170	16	13									27	22
3242.125	15	12									18	15
3254.314			21	17								
3326.907			17	14								
3341.967	18	15	18	15							23	19
3349.043							17	14			20	17
3361.327			8	7			16	14				
3372.901			10	9			18	15				
3380.397			19	16								
3900.681			17	17								
3904.926			15	15								
3913.609			15	15								
3924.673			13	13								
3930.022			9	9								
3947.918			10	10								
3948.818			9	9	19	19						
3956.476			11	11	13	13					18	18
3958.355					16	16						
3981.917					18	18						
3989.912					15	15					16	16
3998.790									15	15	22	22
4009.079			13	13								
4024.726			15	15								
Average	19	16	15	14	16	16	17	14	15	15	21	19

THORIUM.

Wave-length λ	Pressure in atmospheres, followed by $\Delta\lambda$ observed and $\Delta\lambda$ reduced to λ 4000						
			8	9			
4248.1			12 / 11	15 / 14			
4283.7			14 / 13				
4381.6			7 / 7				
4391.3			9 / 8	8 / 7			
4433.2			14 / 13	11 / 10			
4439.3			15 / 14				
4441.1			15 / 14	20 / 18			
4465.5			21 / 19	12 / 11			
4487.7			4 / 4				
4510.7			21 / 19				
Average			13 / 12	13 / 12			

YTTRIUM.

Wave-length λ	Pressure in atmospheres, followed by $\Delta\lambda$ observed and $\Delta\lambda$ reduced to λ 4000								
		$4\frac{3}{4}$	6	7	10	11	$12\frac{1}{2}$	13	
3950.497		5 / 5	7 / 7			15 / 15	17 / 17	14 / 14	
3982.742		4 / 4		4 / 4			17 / 17	18 / 18	
4309.780					11 / 10				
4358.892					17 / 16				
4375.110					21 / 19				
4398.185					23 / 21				
4422.760					20 / 18				
Average		5 / 5	7 / 7	4 / 4	18 / 17	15 / 15	17 / 17	16 / 16	

NOTE.— Several other yttrium lines were measured, and found to agree well with the above.

VANADIUM.

Wave-length λ	Δλ observed and Pressure in atmospheres, followed by Δλ reduced to λ 4000						
			8	10			
3902.399			5 / 5				
3910.984			9 / 9	13 / 13			
3913.0			14 / 14	17 / 17			
3914.5				23 / 24			
3922.560			13 / 13				
3924.8			22 / 22				
3925.4			12 / 12	18 / 18			
3928.1			14 / 14				
3934.2			5 / 5				
3937.7			18 / 18	19 / 19			
3938.3				20 / 20			
3939.5			26 / 26	23 / 23			
3950.4				15 / 15			
3979.6			17 / 17	17 / 17			
3984.5			16 / 16	15 / 15			
3984.7			21 / 21	24 / 24			
3989.0			12 / 12	22 / 22			
3990.712			22 / 22				
3992.971			15 / 15				
3998.9			14 / 14				
4042.8			17 / 17				
4051.204			16 / 16				
4051.491			19 / 19				
4057.2			24 / 24				
4092.821			10 / 10				
4105.318			23 / 23				
4120.6			17 / 17				
4123.539			18 / 17				
4128.251			19 / 18				
4132.100			13 / 13				
4134.589			22 / 21				
Average			16 / 16	19 / 19			

TABLE I.—*Continued.*

URANIUM.

Wave-length λ	Pressure in atmospheres, followed by $\Delta\lambda$ observed and $\Delta\lambda$ reduced to λ 4000						
			9⅝	11	12		
3886.4					12 12		
3993.6			4 4	3 3			
3912.7				5 5			
3916.0				4 4			
3932.2			7 7	11 11	14 14		
3951.5				7 7			
3954.9				6 6			
3982.6				4 4			
3986.0				5 5			
3987.6				13 13			
3989.7				12 12			
4050.3				10 10			
4064.6				13 13			
Average			6 6	8 8	13 13		

ZIRCONIUM.

Wave-length λ	Pressure in atmospheres, followed by $\Delta\lambda$ observed and $\Delta\lambda$ reduced to λ 4000						
				9			
3958.355				13 13			
3999.117				24 24			
4029.796				23 23			
Average				20 20			

TABLE I.—*Continued.*

ZINC.

Wave-length λ	Pressure in atmospheres, followed by $\Delta\lambda$ observed and $\Delta\lambda$ reduced to λ 4000							
		7	8	9¾	11½	12¼	13½	
3075.99		16, 12						
3302.67			36, 30		37, 31			
3345.13			30, 26		38, 32			
3346.04			36, 30					
No series								
Average		16, 12	34, 29		38, 32			
3282.42			33, 27		39, 32			
3303.03			27, 22		44, 37			
3345.62			30, 25		39, 33			
1st subordinate series								
Average			30, 25		41, 34			
3018.50			62, 46					
3035.93			58, 44					
3072.19			64, 49					
4680.317			52, 61	58, 68	51, 60			
4722.341			43, 51	52, 62	63, 77	49, 58	53, 63	
4810.724			47, 56		57, 68	56, 67	62, 74	
2d subordinate series								
Average			54, 51	55, 65	57, 68	53, 63	58, 69	

RELATIONS OF THE SHIFTS TO EACH OTHER AND TO CERTAIN PROPERTIES OF THE ELEMENTS.

As already stated, at least many elements produce lines whose shifts for the same wave-length are quite different, but this difference is by no means a haphazard one. Thus any series (as described by Kayser and Runge) of lines produced by an element gives shifts which, when reduced to the same wave-length, are approximately equal, while the shifts of the series, principal, first and second subordinate, are to each other respectively very nearly as 1 : 2 : 4. As is well known, the lines of any series of a given element are quite similar in general appearance, but very different from those of other series; and the same is true of the several groups of lines, of iron and of copper, for instance, which have such different shifts. In general, then, it appears that lines of the same character of any element, when reduced to the same wave-length, give equal shifts, which, nevertheless, may differ widely from those of a different character, though of the same element.

Lines of the same series, not only of a given element, but also those of different elements, resemble each other closely, and consequently in comparing shifts of lines with other properties of the elements, it is neccessary to confine one's attention, as far as possible, to lines of the same character; and in accordance with this notion the following relations have to do with the shifts of what might be termed the most characteristic lines of the elements, that is, those lines which are the easiest to obtain (at least in the arc spectra) and which produce the sharpest reversals and consequently admit of the most accurate measurements.

On forming, for various elements, the product of the cube root of the atomic volume (*i. e.*, quotient of atomic weight divided by density) and the coefficient of linear expansion of the substance in the solid form, certain numbers are obtained whose ratios are approximately those of the shifts of the lines of the respective elements. This is shown in Table II, in which the atomic volume and coefficient of expansion both refer to 40° C.

TABLE II.

Element	Atomic weight W	$\sqrt[3]{W}$	Atomic volume V	Coefficient of linear expansion a	Temperature of melting point T	$\frac{48600}{T}$	Shift S	$a\sqrt[3]{V}$
				.0000				
Al	27.11	3.00	10.6	2313	1123	43.3	55	50.6
Sb	120.43	4.94	17.9	1692 } 0882 }	710	68	49	{ 43 { 23
As	75.01	4.22	13.2	0559	>773	<63	39	13
Ba	137.43	5.16	36.5	?	748	65	58 } 34 }	?
Be	9.08	2.09	4.9	?	>1270	<39	36	?
Bi	208.11	5.93	21.1	1621	538	90.3	49	44.7
B	10.95	2.22	4	?	?	?	49	?
Cd	111.95	4.82	12.9	3069	593	82	76	75.6
Cs	132.89	5.10	70.6	?	?	?	161	?
Ca	40.07	3.42	25.4	?	>Sr	<Sr	54 } 27 }	?
C	12.01	2.29	3.6	0786 } 0540 } 0118 }	?	?	50 } 0 }	?
Ce	140.20	5.20	21	?	<1273	>38	27	?
Cr	52.14	3.74	7.7	?	?	?	26	?
Co	58.93	3.89	6.9	1236	2070	24	24	23.6
Cb	93.73	4.54	13.0	?	?	?	33	?
Cu	63.60	3.99	7.1	1678	1330	36.5	33	32.5
E	166.32	5.51	?	?	?	?	47	?
Ge	72.48	4.17	13.2	?	?	?	44	?
Au	197.23	5.82	10.1	1443	1310	37	40	67
In	113.85	4.85	15.3	4170	449	108.3	88	103.5
Fe	56.02	3.83	7.2	1210	2080	23.3	25	23.3
La	138.64	5.18	22.5	?	>710	<69	32	?
Pb	206.92	5.92	18.1	29.24	605	80.3	60	76.9
Li	7.03	1.92	12.9	?	453	107	85	?
Mg	24.28	2.90	13.9	2694	1023	47	62 } 44 }	65
Mn	54.99	3.80	6.9	?	2170	22.9	33	?
Hg	200.00	5.85	14.1	6000	233	209	81	145
Mo	95.99	4.58	11.1	?	?	?	40	?
Ndi	140.80	5.20	?	?	?	?	11	?
Ni	58.69	3.89	6.7	1279	1870	26.5	28	24
Os	190.99	5.76	8.5	0657	2770	17.5	17	13.4
Pd	106.36	4.74	9.2	1176	1775	27.4	27	24.7
Pt	194.89	5.80	9.1	0899	2050	23.7	20	18.5
K	39.11	3.39	45.4	8415	335	145	132	300
Rh	103.01	4.69	8.6	0850	2270	21.4	25	17.4
Rb	85.43	4.40	56.1	?	311	156	132	?
Ru	101.68	4.67	8.4	0963	2070	23.5	28	20
Sc	44.12	3.53	17 (?)	?	?	?	24	?
Si	28.40	3.05	11.4	0763	?	?	43	17
Ag	107.92	4.76	10.2	1921	1230	39.5	39	42.2
Na	23.05	2.85	23.7	7105	369	132	108	204
Sr	87.61	4.44	34.9	?	>Ba	<Ba	70 } 35 }	?
Ta	182.84	5.68	16.9	?	?	?	17	?
Tl	204.15	5.89	17.2	3021	563	86.3	102	78
Th	232.63	6.15	20.9	?	?	?	18	?
Sn	119.05	4.92	16.3	2234	503	96.6	55	50.6
Ti	48.15	3.64	13 (?)	?	?	?	22	?
W	184.83	5.70	9.6	?	?	?	19	?
U	239.59	6.21	12.6	?	?	?	11	?
V	51.38	3.72	9.3	?	?	?	25	?
Y	89.02	4.47	?	?	?	?	15	?
Zn	65.41	4.03	9.1	2918	676	71.9	57	61.2
Zr	90.40	4.49	21.7	?	?	?	28	?

and the shift to a pressure of twelve atmospheres and wave-length 4000.

A similar expression is used by Raoul Pictet in his formula for deducing the melting points of the metals. He finds that the continued product of the absolute melting point, the coefficient of linear expansion of the substance in the solid state and the cube root of the atomic volume is nearly the same for all metallic elements except antimony and bismuth. Table II also shows the relation between Pictet's results and the shifts of the lines by giving the quotients obtained by dividing a constant, namely 48600, by the absolute melting points of the elements. The number 48600 was chosen to reduce his results to numbers comparable with the shifts at twelve atmospheres, that of iron being made to coincide with the value given by the product of its coefficient of linear expansion by the cube root of its atomic volume. It appears that the shifts are about as near the calculated values as are the melting points, and consequently in most cases the product of the shift by the absolute melting point is nearly constant; or what amounts to the same thing, the shift is inversely proportional to the absolute temperature of the melting point.

DESCRIPTION OF TABLE II.

The first column gives the symbols of those elements some of whose lines have been examined. Under W are their atomic weights, and in the next column, marked $\sqrt[3]{W}$ the cube roots of these weights. Under V are the atomic volumes at 40°C. and under a the coefficients of linear expansion at the same temperature. The column marked T gives the absolute temperatures of the melting points, and that marked $\frac{48600}{T}$ the quotients indicated. Under S are the shifts at twelve atmospheres and wave-length 4000, and the last column, marked $a\sqrt[3]{V}$, gives the product of the coefficient of linear expansion and cube root of atomic volume multiplied by 10^6. The atomic weights are based on the assumption that the atomic weight of oxygen is 16, and are taken from a special Smithsonian publication (*Constants of Nature*, Part

V, Clark) for 1897. The atomic volumes and melting points are taken from Nernst's *Theoretical Chemistry*, and the coefficients of expansion from the *Physikalish-Chem. Tabellen von Landolt una Börnstein*.

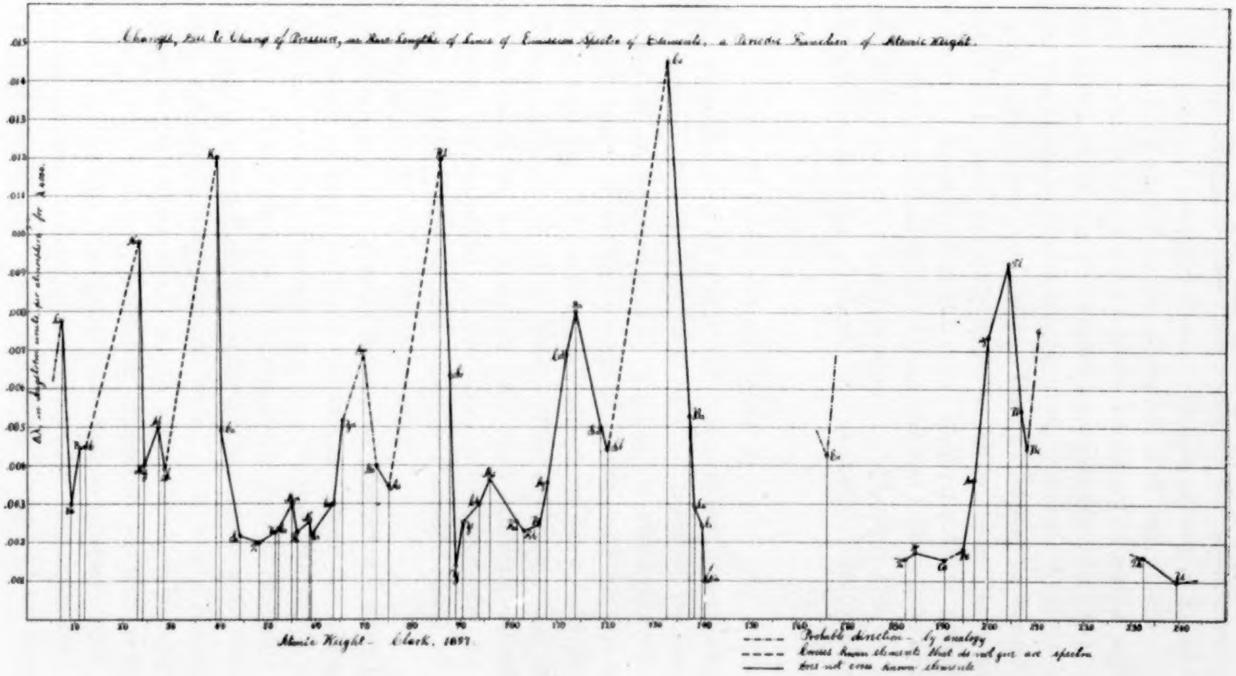
In a few cases, more recent and possibly more accurate values of the above constants have been obtained, but the differences are not sufficient to materially affect the calculated shifts.

Like many other properties of the elements, the shift of their spectral lines is a periodic function of atomic weight, as is evident from the line of shifts as plotted in Plate XVIII, in which the abscissæ are atomic weights and the ordinates the shifts per atmosphere of the lines of the given elements. The maxima fall, as do those of atomic volume, on the alkali metals, lithium, sodium, potassium, rubidium, and cæsium. In the case of those elements that have two or more groups of lines of different shifts, the group giving the best lines was always the one selected. Thus for the alkalies the lines selected are those of the principal series, which are by far the best lines for measurement of these elements, and besides, by selecting lines of the same series, it is possible to compare the shifts of the lines of the alkali metals, all of which belong to the same family of elements, or Mendelejeff group. For similar reasons lines of the second subordinate series were selected in the case of zinc, cadmium, and mercury. For calcium the *g* group of lines was selected, and the corresponding groups for strontium and barium. A different selection of lines where possible would not alter the periodic nature of the curve, but simply increase or decrease the values of the maxima and minima.

As shown by the results tabulated in Table II, those elements, such as sodium, potassium, indium, thallium, cadmium, and others, which have very large coefficients of linear expansion, have also very large shifts, and the converse is equally true.

Another and rather simple relation is this: The shifts of the spectral lines of similar elements, in the main those of the right or left half, as commonly tabulated, of a Mendelejeff group, are

PLATE XVIII.



PRESSURE SHIFTS OF SPECTRAL LINES AS A PERIODIC FUNCTION OF ATOMIC WEIGHT.

generally proportional to the cube roots of the atomic weights of the elements that produce them. This is shown in Table III, in which it will be seen that the observed and calculated values agree quite closely except in very few cases. The single carbon line shifts about twice the calculated amount, and the lines of a few other elements—platinum, osmium, yttrium, thorium, tantalum, and tungsten—only about half as much as would be expected. It is just possible that in these cases lines comparable to those measured of the other elements have not been selected. The lines measured of neodymium and uranium shift much less than the calculated amounts. Whether this is true of all the numerous lines of these two elements I am unable to state.

DESCRIPTION OF TABLE III.

Each horizontal row of Table III contains first the symbol of a certain element, followed by the observed shifts of its lines in thousandths of an Ångström unit for twelve atmospheres and wave-length 4000; then the symbol of an element of the same group followed by the calculated shift of its lines, and finally the observed shift of the same lines. The shifts marked *standard* are assumed to be correct, and each marked *calculated* deduced from the standard of the same horizontal row on the assumption that they are to each other as the cube roots of the atomic weights of the respective elements.

In determining the groups of similar elements I have been guided in some measure by their spectra, and since the grouping adopted is not exactly, though very nearly, that which is commonly made, I have thought it necessary to give it in Table IV. It will be seen that sodium, for instance, is classed with lithium, potassium, rubidium, and cæsium, which it strongly resembles spectroscopically, rather than, as is often done, with copper, silver, and gold, which it does not resemble in this respect. Again, and for similar reasons, magnesium is classed with calcium, strontium, and barium, rather than with zinc, cadmium, and mercury. In no case, however, is there a change of an element

TABLE III.

Showing shifts in thousandths of an Ångström unit for twelve atmospheres and wave-length 4000.

Standard		Calculated		Observed
Cs	161	Li	60	85
Cs	161	Na	90	108
Cs	161	K	109	132
Cs	161	Rb	139	132
Cu	33	Ag	39	39
Cu	33	Au	48	40
Ca	54 27	Mg	46 23	44 30
Ca	54 27	Sr	70 35	65 37
Ca	54 27	Ba	81 40	58 34
Zn	57	Be	30	36
Zn	57	Cd	68	76
Zn	57	Hg	83	81
La	32	Y	28	15
La	32	Sc	22	24
Al	55	B	40	49
Al	55	In	89	88
Al	55	Tl	106	102
Ti	22	Zr	26	28
Ti	22	Ce	30	27
Ti	22	Th	35	18
Sn	55	C	26	50
Sn	55	Si	34	43
Sn	55	Ge	47	44
Sn	55	Pb	66	60
V	25	Cb	28	34
V	25	Ndi	35	11
V	25	Ta	38	17
Bi	49	As	35	38
Bi	49	Sb	41	49
Bi	49	E	45	47
Cr	26	Mo	32	40
Cr	26	W	40	19
Cr	26	U	43	9
Fe	25	Ru	36	28
Fe	25	Os	38	17
Ni	28	Pd	34	27
Ni	28	Pt	42	20
Co	24	Rh	29	25

from one to another of the Mendelejeff groups, and besides the right and left halves, as usually tabulated, are in the main retained, the only changes being in the case of elements of small atomic weights, which have many properties in common with the

elements of each half of their several groups, and which are often regarded as common to the two halves. Thus, as just stated, magnesium is placed with calcium, strontium, and barium, because spectroscopically it resembles them rather than zinc, cadmium, and mercury, though it has so many properties in common on the one hand with those of calcium, strontium, and barium, and on the other with those of zinc, cadmium, and mercury, that it might very well be classed with either group.

TABLE IV.

Showing the adopted grouping of similar elements.

Group I		Group II		Group III		Group IV		Group V		Group VI		Group VIII		
Li Na K Rb Cs	Cu Ag Au	Mg Ca Sr Ba	Be Zn Cd Hg	Sc Y La	B Al Ga In Tl	Ti Zr Ce Th	C Si Ge Sn Pb	V Cb Ndi Ta	As Sb E Bi	Cr Mo W U		Fe Ru Os	Ni Pd Pt	Co Rh Ir

SUMMARY OF RESULTS.

The following list of relations between the shifts of spectral lines, the conditions under which they are produced and the properties of the elements producing them is probably quite imperfect; some of them may be more or less accidental, and in all probability others quite as important have been overlooked. However, I have not searched at all carefully for such relations, and the present list contains only those which presented themselves from time to time during the investigation, and which I trust may be of service to anyone who attempts an explanation of the observed phenomena.

These relations are:

1. Increase of pressure causes all isolated lines to shift towards the red end of the spectrum.

2. This shift is directly proportional to the increase of pressure.

3. It does not depend upon the partial pressure of the gas or vapor producing the lines, but upon the total pressure.

4. The shift of the lines seems to be nearly or quite independent of temperature.

5. The lines of bands (at least of certain "cyanogen" and aluminium oxide bands) are not appreciably shifted.

6. The shifts of similar lines of a given element are proportional to the wave-lengths of the lines themselves.

7. Different series of lines (as described by Kayser and Runge) of a given element are shifted to different extents. When reduced to the same wave-length these shifts are to each other approximately as 1 : 2 : 4, respectively, for the principal, first and second subordinate series.

8. Similar lines of an element, though not belonging to a recognized series, are shifted equally (when reduced to the same wave-length), but to a different extent than are those unlike them.

9. Shifts of similar lines of different substances are to each other, in most cases, inversely as the absolute temperatures of the melting points of the elements that produce them.

10. The shifts of similar lines of different elements are to each other approximately as the products of the coefficients of linear expansion and cube roots of the atomic volumes of the respective elements (in the solid state) to which they are due.

11. Analogous or similar lines of elements belonging to the same half of a Mendelejeff group shift proportionately to the cube roots of their respective atomic weights.

12. The lines of those substances which, in the solid form, have the greatest coefficients of linear expansion have the greatest shifts. The converse is also true.

13. The shift of similar lines is a periodic function of atomic weight, and consequently may be compared with any other property of the elements which itself is a periodic function of their atomic weights.

The results of this investigation can be fairly well expressed by the simple equation.

$$\Delta \lambda = \alpha \beta \lambda (p - p_0)$$

when $\Delta \lambda$ is the increase of wave-length λ of any given line produced by the increase of pressure $p - p_0$, β a constant for any

series of lines and α a constant for any element. That is β for any series of a given element is the same as β for the corresponding series of any other element, while α for any series of a given element is the same as for α for any other series of the same element. If we write β_0 for the principal series, β_1 for the first subordinate and β_2 for the second subordinate, then, approximately, $\beta_0 : \beta_1 : \beta_2 = 1 : 2 : 4$.

By suitably choosing β, α may be replaced in most cases by $\frac{1}{T}$, where T is the absolute temperature of the melting point, or again by $\epsilon \sqrt[3]{V}$ where ϵ is the coefficient of linear expansion of the substance in the solid state and V the atomic volume (the objection to these expressions comes from the fact that for many elements neither T nor ϵ is known); or finally, for either half of any Mendelejeff group, by $\sqrt[3]{W}$, where W is the atomic weight. From this last expression it is evident that α, and therefore $\Delta\lambda$, is a periodic function of atomic weight.

The observations upon which the above conclusions are based, though very numerous, are by no means as complete as could be desired, and I feel quite sure that a more searching examination of a larger number of lines, probably at considerably increased pressures, would add materially to our knowledge of the interesting relations between the spectral lines and the conditions under which they are produced.

DISCUSSION OF THE RESULTS.

How to interpret the shifts of the lines, their relations to each other and to other properties of the elements that produce them is not very evident. However, on any theory of the emission of waves the wave-length increases with increase of the linear dimensions of the segment or portion of matter producing them. Consider therefore an isolated body, to be definite a rectangular bar of steel say, producing vibrations. Waves of different lengths may be given off simultaneously, but the length of each will be proportional to the length of the segment producing it. If the linear dimensions of the bar be increased, the other properties

remaining unchanged, the wave-length of each set of vibrations will be correspondingly increased, the total increase in each case being proportional to the wave-length itself. Now suppose the bar in the midst of a great number of others of either the same or of different material and all moving at random with considerable velocity. Many collisions will take place ; part of the energy of the system becoming internal energy of the bar in question and thereby increasing its linear dimensions and consequently the wave-lengths of its vibrations. Further, the more numerous the bars in a given space the more frequent in the same proportion will be the collisions and therefore the greater will become the internal energy of the bar, its linear dimensions and the wave-lengths of its vibrations. Again, of two bars under the supposed conditions, that one which has the greater coefficient of linear expansion will suffer the greater change in the lengths of its waves.

Since, at any given temperature, the coefficient of expansion of a bar of metal remains constant, so far as is known, no matter how small the bar, and since also the coefficient of expansion of a porous bar, or one bored to any extent and in any direction, is the same as that of a solid bar of the same substance, it would seem on pushing these ideas to the limit, that the coefficient of expansion of a substance in the solid form is also more or less nearly a measure of the expansion of its smallest parts or molecules. Nor does it seem unreasonable to suppose, when these molecules are moving rapidly and in the midst of many others, as they are when the substance is in the form of a gas, that their collisions would lead to an increase of the internal energy of the molecules themselves and consequently to their expansion and to an increase in the wave-lengths of their vibrations. At any rate if the particles producing light-vibrations have properties like those of appreciable masses of the same substance, then the above considerations in regard to the steel bar offer a possible explanation of many, and probably the most important, of the observed facts in regard to the shifts of the spectral lines. In this way is explained why the wave-lengths should always increase with

increase of pressure, and why it is independent of the partial pressure of the gas to which the lines are due. It is also evident that the shifts of the lines should be proportional to their wave-lengths, and greatest for those substances which have the greatest coefficient of linear expansion. This idea offers at least a partial explanation as to why the shifts of the lines should be, as experiment shows them, practically independent of temperature when the pressure is kept constant, since in this case the greater velocity of the particles due to increase of temperature is offset in a measure by the corresponding rarefaction, so that the increase of internal energy of the molecules *due to collisions* may be but slightly, if at all, changed by change of temperature alone. Still an increase of the temperature alone almost certainly increases the amplitudes of the vibrations—the lines become more intense—and it is difficult to see why the internal energy of the molecule should not at the same time be so increased as to make its linear dimensions greater. This, however, it seems would make the wave-lengths greater, a result that is not in accord with experiment.

Why different series of lines of the same element should shift differently is far from evident, and it is with the greatest hesitation that I offer the slightest suggestion in regard to it.

Conceivably the vibrating particles may expand differently in different directions, or possibly the different series of lines may be due to entirely different molecular complexes of very different coefficients of expansion. This latter idea seems to be supported in a measure by the following considerations: The melting point of a substance and its coefficient of expansion both appear to be in some sense inversely proportional to the ability of its particles to resist external influences; and those molecular complexes least capable of resisting external influences may, therefore, at the temperature of the electric arc, be subject to dissociation and possibly to other changes as well. Now it happens that those substances which furnish clearly marked series of lines, as do sodium, potassium, cadmium, mercury, and others, are those whose melting points are among the lowest and

whose coefficients of expansion are the greatest. Should dissociation take place, it is clear that the dissociated parts must be either less subject to external influences or else less powerfully acted upon than are the undissociated parts, else they too would still further dissociate, which process evidently stops somewhere. In either case a smaller coefficient of expansion of the parts might be expected than is that of the undissociated portions. Again it seems but natural to suppose the dissociation taking place along planes of symmetry, should such planes exist, or at least in a manner that would leave the parts with linear dimension approximately but half those of the original whole. Supposing, then, the coefficients of expansion, or better possibly, the amounts of energy used in producing expansion of the parts to be such as to cause these coefficients to be to each other as the linear dimensions of the particles themselves, we have at once an explanation of the 1 : 2 : 4 relation of the shifts of the series.

This also suggests a possible explanation of the fact that analogous elements, as rubidium and cæsium, tin and lead, and others give lines that shift proportionately to the cube roots of their atomic weights, since (if the atoms are of about the same density) the cube roots of their weights are to each other as their linear dimensions. Or, on the other hand, if, as seems possible, the shifts of the lines are to each other as the linear dimensions of the particles to which they are due, it follows that analogous elements—elements of the same half of a Mendelejeff group—differ from each other in the main because of the difference in the linear dimensions of their atoms.

Again I wish to say that these suggestions are offered with the greatest hesitation (the assumptions made are not yet justified), and only with the hope that they may be of some service in mapping out further investigations along this line of spectrum analysis.

HISTORY OF THE PRESENT INVESTIGATION.

The present investigation was begun in February 1895 by Dr. J. F. Mohler and myself, and continued as joint work till

December of the same year. Our accumulated results on twenty-three elements were then published in the ASTROPHYSICAL JOURNAL, December 1895, and likewise, though in a much less complete form, in the Johns Hopkins University *Circulars* for February 1896. A preliminary account of it had also been given by Dr. Mohler at the Springfield (1895) meeting of the American Association for the Advancement of Science.

Dr. Mohler then examined alone the effect of very low pressures on a few of the elements whose behavior at high pressure was already known, and published his results in the ASTROPHYSICAL JOURNAL of October 1896, and I, working independently, extended the examination at high pressures to a number of additional elements, the results of which appeared in the November 1896 number of the same JOURNAL.

The present paper, though including all that has been done in this line, except Mohler's work at low pressure, contains much that is new—the history, so far as there is one, of the subject; the behavior of several additional elements (the metallic elements are now practically exhausted); the behavior of different groups of analogous lines, like those of copper and iron, and in particular the different series of lines as furnished by lithium, sodium, zinc, and others; and also a fuller study of several of the elements, iron, zinc, copper, cadmium, among others previously studied by Mohler and myself.

DESCRIPTION OF PLATE XVII.

Some idea of the effect of pressure on spectral lines may be got from the accompanying plate, which is taken from a few of the negatives obtained during the course of my work. I shows the pair of sodium lines λ 3302.504 and λ 3303.119. The inner portion was taken at a pressure of eight and one-half atmospheres and the outer portion at one atmosphere. On the outside the unsymmetrical nature of these lines is clearly seen, the spreading being to the violet; yet, as shown by the plate, the lines under pressure are greatly shifted towards the red of the spectrum. II is from the New Concord meteorite, the inner portion being

taken at a pressure of twelve atmospheres. This shows several of the iron lines that shift to a greater extent than do others of the same element. It shows also a portion of a cyanogen band, whose lines are not appreciably displaced. Further, it shows that the lines under pressure reverse more readily than do the same lines at normal pressure. III, the inner portion of which was formed at a pressure of seven atmospheres, shows the great difference in the shifts of two classes of copper lines. In this case the plate was taken from an arc formed between a copper and a brass rod, the brass forming the lower and positive pole. IV shows the potassium lines λ 4044.294 and λ 4047.338, which are greatly shifted. The small line between them is due to iron. The pressures were for the outsides and the middle, one and eight and a half atmospheres respectively. V gives the rubidium lines (in the cyanogen band), the shifts of which are very evident. In this case the pressures were also eight and a half and one atmospheres. In fact, V and IV are but different portions of the same plate.

In closing I wish to thank Professor Rowland and Dr. Ames, under whose direction this work was conducted, not only for their assistance every time it was needed, but also for the thoroughly kind and helpful manner in which it was invariably given.

My thanks are also due to Mr. L. E. Jewell for the willingness with which he often brought his extensive knowledge of the spectra of the elements to my aid.

THE NEW SERIES IN THE SPECTRUM OF HYDROGEN.

By J. R. Rydberg.

In the April number of this Journal, page 243, Professor Kayser expresses the opinion that Professor Pickering is wrong in representing the newly discovered hydrogen series in the spectrum of ζ Puppis by the same formula as the old one.

Although I can subscribe without hesitation to the opinion of Professor Kayser as to the relation of the two series, yet the reasons he adduces to confirm his conclusion do not seem quite sufficient. If it should turn out to be a fact that the two series can be united into a single one, then, indeed, strong arguments must be advanced to dispute the correctness of such an arrangement.

The first condition to be fulfilled, before we draw any conclusion from the analogy with other spectra, is to obtain a rigorous proof that the two series have a common limit. For this purpose I have calculated the two series independently of each other, after reducing the wave-numbers to vacuum.

The formula of Balmer for the old series becomes then with Professor Rowland's values

$$n = 27418.75 - \frac{109675.00}{m^2}$$

where n is the wave-number and m the series-number of a line; and my general approximate formula

$$n = n_0 - \frac{109675.00}{(m+\mu)^2}$$

gives for the new series, using the most complete of the observed series for ζ Puppis,

$$n = 27418.79 - \frac{109675.00}{(m+0.500737)^2}.$$

The corresponding values in air (16°, 760^{mm}) are :

m	5	6	7	8	9	10
λ obs.	4201.6	4026.5	3924.9	3858.6	3817.2	3783.4
λ calc.	4201.54	4027.31	3925.18	3859.76	3815.17	3783.35
diff.	+0.06	−0.81	−0.28	−1.16	+2.03	+0.05

Thus the limit n_0 coincides exactly in the two series, and we evidently obtain the formula of Professor Pickering by inserting in place of 0.500737 the approximate value 0.5, and then substituting $\frac{1}{2}m$ for m.

From this we see first of all that the condition for the correspondence of the series, as nebulous and sharp series of hydrogen, are fulfilled as exactly as possible, if we consider their different intensity and sharpness, as pointed out by Professor Kayser. Consequently we are perfectly justified in comparing these series with those known in other spectra. Secondly, we find that the formula of Professor Pickering contains the two given here and differs from them only by giving other numbers to the lines and representing the two series as one.

Which of these two methods is to be considered the correct one can easily be decided by reference to the existing analogy with other spectra, seeing that we have for some elements hitherto examined the following values of the constant μ:

Element	Nebulous series	Sharp series
H	1.000000	0.500737
Pa	0.997273	0.858110
He	0.996084	0.701464
Li	0.998063	0.597337
Na	0.988436	0.649840
Zn	0.905336	0.269148
Cd	0.906478	0.327899
Ag	0.982165	0.447358
Cu	0.975792	0.399765

If the two series were related in the way indicated by the formula of Professor Pickering, the values of μ for the same element ought to differ by a quantity approaching 0.5, so that the terms of one series would fall half-way between the terms of the other. But, as we see from the table, these differences vary in an irregular manner. Therefore hydrogen would be the only known element by which the two series could be thus united in one formula. It is worthy of notice that the formula of Kayser and Runge

$$n = A - Bm^{-2} - Cm^{-4}$$

does not permit a comparison of this kind between the series.

Another argument, not yet mentioned, against the combination of the two series, which would be sufficient to decide the question, can be derived from the analogy with other spectra. For, as I have shown explicitly,[1] the constituents of the doublets and triplets of the nebulous series are built up after exact rules, and form new doublets and triplets of a higher order, while the constituents of the sharp series, so far as we know, are simple lines. Evidently it would be absurd to unite these two kinds of lines in one series, where the terms would be alternately simple and alternately double and triple, as would be a necessary consequence if we should follow the analogy given by the formula of Professor Pickering.

We arrive, therefore, at the definitive conclusion that *the two series of hydrogen are to be represented by two distinct formulæ,* even if it may be possible to unite them with great approximation in a single equation.

Hitherto the spectrum of hydrogen has appeared through its simplicity to differ from all others, and I have also adduced this circumstance among the many which give to hydrogen a quite peculiar place in the system of elements. Through the discovery of the new series the analogy between this spectrum and the others is evident; but there remains still the exceptional simplicity in the values 1 and ½ for the constants μ of the nebulous and of the sharp series. However, as the analogy ought to hold good in all details, we are compelled to adopt the assumption that the lines of hydrogen are double, just as are those of other elements. In this connection we may recall an observation made by Professor Michelson in his researches on close doublets by the interference method, from which he found that *Ha* actually has two components of nearly the same intensity, quite as we should expect from the monatomicity of hydrogen. Now, if this be the case, it is evident that the formula of Balmer will lose its quality, hitherto unparalleled in

[1] *Wied. Ann.*, **50**, 629, 1893.

science, to represent with absolute exactness a series of quantities given by nature. The formula is to be divided into two, with different values for the limit n_0, but with the same value for the constant μ, which very likely will differ a little from the exact values 0 or 1.

At all events we can now, without any risk of mistake, venture to compute the second part of the sharp series of hydrogen, the so-called *principal series*. Then, as I have pointed out already in my general exposition of the constitution of line spectra[1] and have afterwards tried further to confirm,[2] there can be no doubt that these series are really parts of a single group of lines with two variable integral parameters, the general formula of which can be written approximately[3]

$$\frac{n}{109675.00} = \frac{1}{(m_1 + \mu_1)^2} - \frac{1}{(m_2 + \mu_2)^2},$$

m_1 and m_2 being the parameters, and μ_1 and μ_2 constants. In the present case the formula becomes

$$\frac{n}{109675.00} = \frac{1}{(m_1 + 1)^2} - \frac{1}{(m_2 + 0.5)^2},$$

where for the sharp series m_1 is always unity, and m_2 is variable and can assume the values 1, 2, 3, 4, ... ; for the principal series, on the contrary, $m_2 = 1$, while m_1 varies. By giving m_1 or m_2 the constant values 2, 3, 4, ... and then varying m_2 or m_1 respectively, other series are formed. Negative values of n have the same meaning as positive ones.

On computing from the above formula the values of n for the principal series we find the following lines:

m_1	1	2	3	4	5
n	21325.69	36558.33	41889.75	44357.44	45697.91
λ	4687.88	2734.55	2386.50	2253.74	2187.60

[1] "Recherches sur la constitution des spectres d'émission," etc. *K. Svenska Vetensk. Akad. Handl.*, **23**, No. 11, 1889.

[2] This JOURNAL, **4**, 91, 1896.

[3] "Recherches," etc., p. 64. This law is also given explicitly in the abstracts of this memoir in *Zeit. Phys. Chem.*, *C. R.*, and *Phil. Mag.* for 1890, so that it seems impossible for Professor Schuster to base his claim to a second discovery on the law not having been sufficiently published.

The values of n are calculated for vacuum, but the values of λ are reduced to air (16°, 760^{mm}) in order to correspond to the observed values.

A glance at the above numbers shows at once that only the first line is likely to be found in star spectra, as all the others are situated in the region cut out by the absorption of the atmosphere. This line, which would correspond to the red line of Li ($\lambda = 6708.2$) and to the two D lines of Na, ought to possess an extraordinary intensity, exceeding by far that of all the other lines of hydrogen in the visible spectrum.

These conclusions are confirmed in every respect, if we consider the spectra of stars of the fifth type. In the *Annals of the Harvard College Observatory*, **28,** Part 1, p. 48 ("Spectra of Bright Stars," by Antonia C. Maury and E. C. Pickering) we find in Table III, in the column under "Curve," the following lines with their intensities:

	λ	i		λ	i
H [D_1 7]	3889	1	H [S_1 5]	4200	3
H [S_1 7]	3926	1	H [D_1 4]	4340	3
H [D_1 6]	3970	1	H [S_1 4]	4544	2
H [S_1 6]	4026	1	....	4614	2
....	4059	4	H [S_1 1]	4688	10
H [D_1 5]	4102	5	H [D_1 3]	4862	1

The designations in the first column correspond to the system introduced for other spectra.[1] D is a line of a nebulous (diffuse) series, S a line of a sharp series, the numbers give the values of m.

As we see, *all the known lines of hydrogen are surpassed in intensity by the line 4688, which corresponds almost exactly to the computed value 4687.88 and which we can, with full certainty, indicate as the first line of the hydrogen spectrum, being at once the first term of the principal and of the sharp series.*[2]

Of the remaining lines in the spectrum of hydrogen H [S_1 3]

[1] "Recherches," chap. v, p. 76.

[2] There is a line at λ 4687 n the spectra of several nebulæ.—EDS.

is, no doubt, the line 5412.4 quoted by Professor Pickering from the measurements of Professor Campbell, and H [D_1 2] is $H\alpha$. As to H [S_1 2] the formula gives $\lambda = 10128.17$, as already calculated by Professor Pickering, and there can be no doubt that the line is to be found in the spectra of stars of the fifth type, the adjacent terms H [S_1 1] and H [S_1 3] being certainly known. But the first term of the nebulous series, H [D_1 1], should have $n = 0$, $\lambda = \infty$, if the formula is exact, and then no such line would exist. If, on the contrary, the formula of Balmer is only approximate, we will have a doublet with a greater wave-length than any line we know or can at present infer from any spectrum hitherto investigated.

Besides, the close agreement between the observed and the computed values of H [S_1 1] shows clearly that $\mu = 0.5$ is to be preferred to $\mu = 0.500737$, which would give $\lambda = 4698.43$ instead of 4687.88.

LUND, July 28, 1897.

ON TRIPLETS WITH CONSTANT DIFFERENCES IN THE LINE SPECTRUM OF COPPER.

By J. R. RYDBERG.

THE experience acquired in the examination of a great many spectra shows that the lines which can be arranged in series according to rules already discovered, form but a small portion of the whole number of lines, which we can obtain under different conditions from the same incandescent gas. In the spectra of the biatomic elements I have pointed out, in addition to the ordinary series, regular groups of doublets and triplets closely agreeing in their constitution with the doublets and triplets by which the known series are formed.[1] Here we find constant differences of wave-numbers, nebulous and sharp, simple and compound triplets, and we have strong reasons to assume that these can be united in series analogous with those already known. However, this analogy does not seem equally clear, when we consider the spectra of the heavy metals with their immense numbers of lines, regarding the connection of which we have hitherto been completely in the dark. This applies, for instance, to the spectra of the Fe group, of the Pt group, of Cu, Ag, and Au. So far as we know it is not impossible that the constant differences of wave-numbers are limited to the ordinary series, and that the spectra of higher temperatures are built up in quite another way, so that some new key must be found to make a grouping of the lines practicable.

To decide, if possible, this question I have examined the spectrum of Cu after the measurements of Professors Kayser and Runge,[2] and I have found that the law of constant differences holds good even outside the ordinary series. As the lines which I have hitherto been able to arrange are in general very weak, no trustworthy conclusions can be drawn from their relative

[1] "Recherches sur la constitution des spectres d'émission." *K. Svenska. Vet' Akad. Handl.*, **23**, No. 11, 100. *Wied. Ann.*, **52**, 119.

[2] "Über die Spectren der Elemente," **5**, 8–17.

intensities, but it seems not impossible that we have to do with a constitution differing considerably from any that we have previously known. However, we will retain the word "triplets" to designate recurring groups of three lines with constant differences between the wave-numbers.

In my examination of the Cu spectrum I have met with two different groups of triplets. The *first group* gives the differences $\nu_1 = 129.50$, $\nu_2 = 50.58$, and consists of the following complete triplets.[1]

i *n*	ν_1	*i* *n*	ν_2	*i* *n*
(5d) 26599.07	129.46	(4d) 26728.53	50.47	(5d) 26779.00
(5d) 27196.31	130.28	(6d) 27326.59	51.47	(6n) 27378.06
(5d) 27278.80	129.57	(6d) 27408.37	50.65	(5d) 27459.02
(5d) 28196.50	128.10	(4) 28324.60	49.75	(5d) 28374.35
(6) 29443.03	129.47	(4d) 29572.50	50.63	(5d) 29623.13
(3d) 31987.51	129.76	(5d) 32117.27	51.14	(3d) 32168.41

The first, third and fifth of the triplets give almost the same values for ν, which value I consider as normal. In the second and the sixth both ν_1 and ν_2 are greater, in the fourth triplet both are smaller. By analogy this would indicate that the three last-mentioned triplets are really compound, but the components too weak to be observed.

The same differences of wave-frequencies occur also in the following doublets, some of which seem to be intimately connected with adjacent triplets:

i *n*	ν	*i* *n*	*i* *n*	ν	*i* *n*
(2rd) 22026.53	129.76	(5d) 22156.29	(4d) 33083.55	128.89	(4) 33212.44
(5d) 27233.12	129.66	(5n) 27362.78	(5n) 38768.71	130.29	(5d) 38899.00
(5) 30488.83	129.19	(5d) 30618.02	(4) 41663.54	130.07	(2d) 41793.61
(5d) 27620.72	50.52	(6d) 27671.24	(5n) 35885.38	50.55	(5n) 35935.93
(4d) 28297.83	50.46	(4d) 28348.29	(3r) 44185.22	51.21	(4r) 44236.43
(5d) 29392.05	50.98	(6) 29443.03	(5) 45081.39	49.44	(3r) 45130.83
(4d) 30461.99	50.47	(6) 30512.46	(4r) 45833.72	50.26	(4) 45883.98
(5n) 33524.31	50.54	(5n) 33574.85	(6r) 46043.06	50.72	(6) 46093.78

[1] The intensity decreases from 1 to 6; *d* corresponds to "verbreitert," *n* to "sehr unscharf," *r* to "umgekehrt" of Kayser and Runge; the other lines are sharp.

Several of these doublets may be only accidental, without real connection between their components.

The *second group* of triplets has greater differences of wave-frequency, namely, $\nu_1 = 680.19$, $\nu_2 = 212.21$. The following are complete:

i	n	ν_1	i	n	ν_2	i	n
(4)	18001.28	680.29	(5)	18681.57	212.20	(3)	18893.77
(4n)	18546.37	680.33	(4d)	19226.70	212.10	(5d)	19438.80
(4d)	26728.53	679.84	(6d)	27408.37	212.35	(5d)	27620.72
(5d)	26799.00	680.02	(5d)	27459.02	212.22	(5d)	27671.24
(5)	32319.89	679.82	(5d)	32999.71	212.73	(4)	33212.44
(4)	32532.17	680.37	(4)	33212.44	211.03	(5n)	33423.47
(2r)	45153.25	680.47	(4r)	45833.72	209.34	(6r)	46043.06

In the following doublets we meet with only one of the differences in question:

i	n	ν	i	n	i	n	ν	i	n
(5n)	20858.41	680.41	(5n)	21538.82	(6)	29443.03	679.63	(4d)	30122.66
(5d)	23571.45	680.50	(5d)	24251.95	(5)	30488.83	680.13	(4)	31168.96
(5d)	24251.95	679.74	(5n)	24931.69	(4n)	31307.04	680.47	(3d)	31987.51
(5d)	26599.07	679.73	(5d)	27278.80	(5n)	31488.03	680.38	(3d)	32168.41
(6d)	27667.80	680.49	(4d)	28348.29	(4d)	31961.85	680.55	(3)	32642.40
(4n)	21287.38	211.94	(2)	21499.32	(6d)	29238.06	212.51	(5d)	29450.57
(5d)	27196.31	212.06	(6d)	27408.37	(5d)	32045.84	212.54	(4d)	32258.38
(6d)	28196.50	212.03	(4d)	28408.53	(5)	32319.89	212.18	(4)	32532.07
(5d)	28374.35	212.52	(5d)	28586.87	(4n)	44502.20	212.52	(6r)	44714.72

In the triplets and doublets the lines 24251.95, 27408.37, 32319.89, 32532.07, and 33212.44 occur twice.

Between the two groups of triplets we find a connection which I have never observed before. As is to be seen from the following arrangement of the wave-numbers with their differences, we have a group of lines similar to the compound triplets of the biatomic elements; save that here both of the two characteristic pairs of differences occur in simple triplets, while in the other case we have one pair only as peculiar to the spectrum.

(5d) 26599.07	129.46	(4d) 26728.53	50.47	(5d) 26779.00
679.73		679.84		680.02
(5d) 27278.80	129.57	(6d) 27408.37	50.65	(5d) 27459.02
		212.35		212.22
		(5d) 27620.72	50.52	(5d) 27671.24

Indications of such groups are met with more than once, but in general they seem to be incomplete.

Among other differences, which appear very frequently, we find a great many which are a little smaller than the known difference 248.54 of the doublets of the ordinary series. Between 238.5 and 245.5, for instance, there are no less than 42. The numbers do not coincide so closely as in the cases already considered, owing, perhaps, to a constitution of the doublets analogous with that we are acquainted with in the nebulous series, so that constant differences would really exist between one of the lines and a weaker companion, not yet observed, of the other. For example, I will give a few pairs of lines in which the concordant character of the constituents seems to indicate a real connection.

i	*n*	ν	*i*	*n*
(2)	21255.02	244.30	(2)	21499.32
($5d_r$)	39008.56	242.99	($5d_r$)	39251.55
($2d_r$)	41548.60	245.01	($2rd_r$)	41793.61
(2r)	44886.33	244.50	(3r)	45130.89

On placing the means of all the constant differences of Cu together in order of their magnitude we find,

ν_1	ν_2
680.19	212.21
248.54	
129.50	50.58

This group of values of ν evidently resembles the corresponding numbers of the Ca group,[1] but we are unable at present to decide whether the resemblance is real or apparent. I have tried in vain to arrange the new triplets in series. At all events there can be no doubt that the law of constant differences extends beyond the limits of the first discovered series. In another

[1] *Wied. Ann.*, **52**, 126.

respect also the triplets of Cu must excite interest. Hitherto triplets have been recognized only in the spectra of biatomic elements, and the series of Cu supposed to be made up of doublets like those of elements of uneven valency in general. Now the difference between elements of uneven and of even valency is partly smoothed out, triplets having been found in the former group as well as doublets in the latter.

LUND, August 2, 1897.

RÉSUMÉ OF SOLAR OBSERVATIONS MADE AT THE ROYAL OBSERVATORY OF THE ROMAN COLLEGE DURING THE FIRST HALF OF 1897.

By P. Tacchini.

I give below a résumé of the solar observations made during the first half of 1897. The following results have been obtained for the spots and faculæ:

1897	Number of days of observation	Relative frequency		Relative size		Number of spot groups per day
		of spots	of days without spots	of spots	of faculæ	
January	23	13.61	0.00	108.0	70.2	2.9
February	19	9.00	0.00	59.5	74.7	2.6
March	24	8.8	0.04	26.5	62.5	2.8
April	25	10.44	0.12	30.1	67.7	3.3
May	25	5.72	0.20	29.5	55.4	1.6
June	29	3.14	0.31	13.1	48.1	2.0

Sun-spots have thus continued to diminish in number with a secondary minimum in the month of June; in correspondence with this diminution we find an increase in the number of days without spots. In the case of the faculæ we also have a minimum in the month of June, but the difference between this and the preceding series is not so well marked as for the spots.

For the prominences we have obtained the following results:

1897	Number of days of observation	Prominences		
		Mean number	Mean height	Mean extent
January	14	3.71	37.2	2.1
February	15	4.47	33.1	1.5
March	19	5.42	38.3	1.8
April	21	3.86	34.9	1.4
May	23	3.30	33.2	1.4
June	28	4.00	36.6	1.4

It thus appears that even the prominences have experienced a diminution of activity. The highest prominence observed

(January) reached an altitude of 93″, and not a single prominence is worthy of special remark.

For the distribution in latitude of the various phenomena I have obtained the following data, grouped by zones for each quarter:

1897												
Latitude	Prominences				Faculæ				Spots			
	First quarter		Second quarter		First quarter		Second quarter		First quarter		Second quarter	
90°+80°	0.004		0.000									
80 +70	0.008		0.003									
70 +60	0.008		0.003									
60 +50	0.050		0.087		0.000		0.004					
50 +40	0.034	0.381	0.083	0.545	0.000		0.000					
40 +30	0.067		0.056		0.032		0.021					
30 +20	0.050		0.125		0.069	0.403	0.057	0.410	0.000			
20 +10	0.059		0.118		0.127		0.119		0.122	0.367	0.000	0.351
10 + 0	0.101		0.070		0.175		0.209		0.245		0.351	
0 —10	0.156		0.098		0.185		0.246		0.306		0.433	0.649
10 —20	0.169		0.090		0.196		0.193		0.286	0.633	0.216	
20 —30	0.139		0.101		0.100		0.098		0.041			
30 —40	0.038		0.049		0.063	0.597	0.024	0.590				
40 —50	0.063	0.619	0.066	0.455	0.048		0.021					
50 —60	0.042		0.031		0.005		0.008					
60 —70	0.004		0.007									
70 —80	0.008		0.003									
80 —90	0.000		0.010									

While the faculæ and spots show in the two quarters a greater frequency in the southern zones, the frequency of the prominences has been a little greater for the northern hemisphere in the second quarter. Prominences have been seen in almost all the zones, while the faculæ have been confined within the limits $\pm 60°$, and the spots within the zones $(+20° — 30°)$. I have observed no eruptions during the six months, and on the spots I noted merely the reversal of the lines D_1, D_2 on the 14th of January over the great spot near the western limb. One might therefore almost affirm that the constitution of Sun-spots has undergone a change!

ROME, August 20, 1897.

HELIOGRAPHIC POSITIONS. I.

By FRANK W. VERY.

THE study of the solar surface is now and must continue to be a very important department of astrophysics. With the possible exception of the human brain, there is no more wonderful object in nature than the Sun, but only those who, with the aid of a powerful telescope and rare atmospheric quiescence, have been able to gaze for hours on the majestic turmoil of a great Sun-spot in full vigor, can appreciate the fascination which attaches to this study. Such favored ones will recognize that, while the investigation of the intimate surface structure of the Sun by Secchi, Langley, Janssen, Young, and others, together with the consecutive researches of solar history by Carrington, Spoerer, Wolf, De La Rue and Stewart, Tacchini, and the Greenwich observers, have given us a basis of firmly established facts, we have scarcely begun to realize the meaning and the variety of the phenomena involved; and only minute and assiduous study will open the solar secrets further. The beginner in such studies can at first do little more than familiarize himself with the leading facts by actual observation, and verify the conclusions of his predecessors. Even this demands a very considerable outlay of time and patience, and it is for such students that I propose to review the initial steps for obtaining accurate positions on the solar surface. No especial claim is made for originality, but I shall endeavor to gather into one body a considerable mass of scattered information, and, whatever else may be lacking, to at least present this material in an intelligible form. I find that the relative positions of points on the Sun-sphere, although governed by elementary considerations, are sufficiently intricate to be difficult of retention in the memory, and judging from their published figures, many professional astronomers share in my failing. I trust, therefore, that the details given here may not seem superfluous.

In photographing the Sun it will usually be most convenient to adopt a fixed distance for the photographic plate, or at least a distance which varies only with the temperature, and which is determined experimentally as the one giving the sharpest focus with the given optical apparatus and the particular photographic film employed. The diameter of the image will in this case vary from day to day, and each picture must be reduced separately. The labor, however, can be minimized by preparing suitable tables.

Where the method of Sun-drawing upon a solar projection is employed, it is best to adopt a convenient diameter for the solar image of an exact number of scale units (*e.g.*, 20^{cm}, 8 inches, ½ foot, or some such number) for convenience in reduction, adhering to this magnification throughout the entire series. Circles of the adopted size may be drawn in advance upon white paper. The paper is pinned upon a drawing board, sliding on rigid guides attached to the telescope and with its plane perpendicular to the optical axis. The distance of the paper is varied from day to day so that the carefully focused solar image shall fit the circle, giving a constant scale of linear distances at the Sun's surface, though a variable one for angular measures on the celestial sphere. A shield of black cardboard, about three feet square, attached to the telescope, gives sufficient shade to the drawing board if indoors, but in the open air a closed canopy of black cloth may be needed. The position and outlines of spots are traced while the clock-driven telescope keeps the image coincident with the bounding circle of the projection. If a horizontal meridional heliostat is used, it will be necessary to rotate the drawing in the plane of projection about the optical axis, and to work rapidly. This arrangement is consequently ill-adapted to anything but instantaneous photography; and for hand work there are advantages in the coelostat, rotating at half speed and sending the sunbeam east or west.

A series of four double charts in orthographic projection, showing positions of meridians and parallels for every 10°, to 40° north and south latitude, on a scale of five inches to the

solar diameter, has been published by Ball in his *Atlas of Astronomy*.[1] These are really equivalent to eight single charts, since each can be used upside down, and they permit the determination of heliographic positions by inspection to within a degree or two, which in view of the rapid changes in Sun-spots is nearly accurate enough for general positions, but not for the detailed study of the changes in question which are of sufficient interest to be pursued for their own record and elucidation. If greater accuracy should be desired by this method, a series of seven double charts, one for each degree of the varying inclination of the Sun's equator to the ecliptic, might be constructed, and such a series would be sufficient for reading spot-positions in Sun-drawings to something less than a degree, but the method is necessarily of limited value.

Photographs are worthy of more precise treatment, and here, if the image has been projected by an eyepiece, the distortion of the field becomes appreciable. A very good way to determine the correction for distortion is to take chronographic transits of both limbs of the Sun, or of a small spot, across a system of lines, ruled parallel at a distance $s = \frac{1}{4}$, or $\frac{1}{2}$ inch apart, or in general at some interval not far from a minute of arc, and placed at right angles to the solar motion at the distance of the usual projection. We will suppose that one of these lines, which may be called the central line, intersects the optical axis, and the lines to be so equally spaced that the time of passage across the central line coincides with the mean for the symmetrically disposed system. We then take the mean (for both limbs) of the time-intervals from mean passage as determined from the combination of all the transits. The correction for the instrumental variation in the direction of the Sun's path in a meridional heliostat image is scarcely appreciable in so short an interval, and is nonexistent in an equatorial image; but that for the Sun's motion in right ascension must be applied. Five series of double transits of the Sun's limbs across a system of twenty-five parallel lines, or ten series of transits of a spot, will be sufficient. For some purposes

[1] Published by D. Appleton & Co., New York, 1892.

we may wish to state the distortion-correction as a function varying from zero at the center to its maximum value at the edge of the field, or at the radius (R) of the solar image. Here it will be well to make a larger number of observations of the transit across a central interval, or across the two intervals on either side of the central line, the mean (I_1) being assumed to represent the time-interval for an undistorted image, and this, multiplied by the number of spaces to the edge of the solar image (n_R), gives a computed interval,

$$I_{R'} = I_1 \, n_R = I_1 \, \frac{R}{s},$$

whose ratio to the time of passage of the semi-diameter is a number, having an excess above unity which is the distortion at a distance from the center of the field equal to the radius of the Sun's image, the ratio of times being also an expression in terms of the linear radius in this particular case. The distortion may be converted into inches or centimeters on the scale of projection by multiplication by the number of these units in the radius of the solar image, or may be stated in seconds of arc by multiplying by the tabular value of the Sun's radius in seconds (R''). In like manner the distortion at any point in the field (whose radius is r) may be expressed as a percentage of the corresponding radius of an undistorted field (r') by multiplying the mean interval for a central space by the number of spaces to the given radius,

$$I_1 n = I_{r'},$$

and comparing with the observed interval from central passage (I_r). A point in the solar image at the real radius (r') will be found in the distorted image at

$$r = \phi\,(r').$$

In comparing distances and intervals, we see that

$$r : r' = \int_0^{r'} \phi(r')\,dr' : r' = I_{r'} : I_r,$$

that is to say, the greater the distortion or magnification of a given part of the telescopic image, the shorter will be the transit

interval for the corresponding fraction of the normal transit, the times of transit being proportional to the distances. The distortion at the radius r' (expressed as a fraction of the undistorted radius) is:

$$\frac{r - r'}{r'} = \frac{I_{r'} - I_r}{I_r};$$

and the correction to the measured radius (r), required on account of distortion, is:

$$\frac{r' - r}{r} \times r = \frac{I_r - I_{r'}}{I_{r'}} \times r,$$

subtractive if the distortion has increased the size of the image The corrected radial distance is therefore:

$$r' = r + \left(\frac{I_r}{I_{r'}} - 1\right) r = r \times \frac{I_r}{I_{r'}}.$$

Inasmuch as we do not wish to get the final value of the radius in linear units, but as a fraction of the solar semi-diameter $\left(\frac{r}{R}\right)$, and since the time of passage of the semi-diameter is the same in a distorted as in an undistorted image, the correction for distortion, as applied to the ratio $\frac{r}{R}$, is zero at the center of the solar disk and at the limb (it being assumed that the solar image is central in the field) and attains a maximum at a little over half the radius of the disk. For the radius r, the observed interval from central passage, expressed as a fraction of the time of passage of the semi-diameter, is $\frac{I_r}{I_R}$; and since the time-intervals are proportional to the real distances, this is also the corrected value of $\frac{r}{R}$. The ratio of the computed intervals for an undistorted image is the same as the ratio of the distances, or

$$\frac{n_r I_1}{n_R I_1} = \frac{r}{R},$$

and the disagreement of computed and observed intervals is the evidence of distortion. That is to say, the point whose position in the distorted image has the fractional radius $\frac{r}{R}$, will not

exhibit a corresponding ratio of intervals, but will be found in an undistorted image at $\frac{r'}{R'}=\frac{I_r}{I_R}$. The distortion of the fractional radius is therefore

$$\frac{r}{R}-\frac{r'}{R'}=\frac{r}{R}-\frac{I_r}{I_R}.$$

and the correction to be applied to the distorted fractional radius is:

$$\frac{r'}{R'}-\frac{r}{R}=\frac{I_r}{I_R}-\frac{r}{R},$$

or the corrected value of $\frac{r}{R}$ is:

$$\frac{r'}{R'}=\frac{I_r}{I_R}.$$

If, as before, distortion has increased the size of the image, $\frac{I_r}{I_R}>\frac{r}{R}$, or the value of $\frac{r}{R}$, obtained on the erroneous assumption that the image is undistorted, is too small, whence the correction to $\frac{r}{R}$ is positive.[1]

The value of the radius at any point of the solar disk, expressed in seconds of arc, is:

$$r''=\frac{I_r}{I_R}\left(R''\right),$$

and the correction to the radius in seconds (where the measurement has been made on the erroneous assumption of no distortion in the image) is:

$$c''=\left(\frac{I_r}{I_R}-\frac{r}{R}\right)\times\left(R''\right),$$

where, as before, I_R is the observed time of passage of the semi-diameter, and R'' the tabular radius of the Sun in seconds of arc.

The scale-value (V_0'') in seconds of arc per linear unit at the center is:

[1] The accompanying note p. 259 by Mr. F. Slocum, a student at this Observatory will illustrate the application of these principles.

$$V_O'' = m\left(\frac{R''}{R}\right) = \frac{I_1}{I_R} \times \left(\frac{R''}{s}\right),$$

whence, at the center, the multiplier (m) is:

$$m = \frac{I_1 \, R}{I_R \, s};$$

but at any any other point of radius r,[1] the scale-value will vary according to the equation

$$V_r'' = m\left[\frac{1}{R} + \frac{r}{R}\left(\frac{I_r - I_{r'}}{I_{r'}}\right)\right] \times \left(R''\right).$$

The multiplier (m) is a variable, but an approximate average value can be found for it by taking the ratio of computed and observed intervals corresponding to the radius at the center of gravity of the area included in the plotted distortion curve. The values computed in this way may need a little adjustment by comparison of the integrated radii (obtained by summing the scale-value for short intervals) with the true radii,

$$r'' = \frac{I_r}{I_R}\left(R''\right).$$

Having obtained the scale-values in seconds, it is a simple matter to transform them into miles measured at the Sun's surface. The distance to be used is, of course, that from the Sun's surface to the Earth.

Position-angles in either drawings or photographs are readily measured with the protractor, and need only a passing mention.

The correction for distortion of field is not appreciable where no eyepiece is applied, unless the field is much greater than is ever employed in practice; but for projection-images, formed by the aid of eyepieces, the correction is required in accurate work for all but very narrow fields near the optical axis. In work with the position filar micrometer, since the eyepiece is merely used to examine the field of the objective, no correction for distortion is needed. With this instrument, therefore, we may measure spot-positions directly in polar coördinates.

[1] For the particular case of the marginal radius, $V_R'' = V_O'' + \frac{I_R - I_R'}{I_R'}\left(R''\right)$

Sun-drawings made upon paper by the method of projection cannot pretend to very great accuracy, even when most carefully executed. Paper is hygroscopic, and, when placed in a warm solar image, changes its dimensions, even if there be no variation of humidity between the observing and computing rooms. To avoid this buckling and distortion of paper by the Sun's heat, Mr. Carrington made all of his long series of Sun-drawings upon ground glass, with an image eleven inches in diameter, all measurements being made either on the ground glass (coated with whitewash, tinted yellow and backed by dead black paper) or by the method of timing to be described presently. The drawings were transferred to paper for preservation, but no measures of accuracy were made from these paper relics.

The method of timing "grew out of a somewhat rude notion of making the disk of the Sun its own circular micrometer." A pair of cross-wires being placed at right angles to each other, approximately 45° to the north and south line, and at the focus of the telescope, the times of transit of the Sun's limbs, and of a spot, may be taken to determine the direction of the Sun's path and the position of the spot. For example, if the experiment be made at the solstices, the Sun will move along a declination, parallel, and assuming that the Sun's center does not pass directly over the intersection of the wires, or that these are not exactly 45° to the north and south line, we shall have first contact of the preceding limb at one of the wires, which may be called A, the other being B. Let the times of contact in their order be A_1, B_1, A_2, B_2. Then calling α the angle made by wire A with a normal to the Sun's path, or with an hour-circle if the deviation of the Sun's path from an east and west line, and the variation of the refraction be neglected,

$$\tan \alpha = \frac{A_2 - A_1}{B_2 - B_1},$$

During the half year preceding the summer solstice, a line through the intersection of the cross-wires, at right angles to the Sun's diurnal path, will incline towards the east of north, and towards the west in the second half of the year. Hence a cor-

rection for the deviation of the Sun's path must at such times be applied to α, which is:

$$\kappa = \frac{\Delta\delta}{15 \cdot \sin p \times 3600 \times \sin 1''},$$

where $\Delta\delta$ is the Sun's hourly increment of declination given in seconds of arc, and p is the north polar distance of the Sun. The position-angle of wire A is therefore $\alpha \pm \kappa$, the sign of κ being positive from winter to summer solstice, and negative the rest of the year.

Mr. Carrington has given[1] a formula by which the angle α may be corrected for differential refraction. Calling α' the corrected angle,

$$\tan \alpha' = \tan \alpha \cdot \sqrt{\frac{1 - e^2 \cdot \cos^2 (\alpha + s)}{1 - e^2 \cdot \sin^2 (\alpha + s)}},$$

in which s is the Sun's parallactic angle, positive for west hour-angles, and negative for east, while e is the eccentricity of the solar disk due to refraction. The correction is of troublesome application, and may be safely neglected in most observations, since the atmospheric tremor will seldom permit work at the low altitudes where the consideration of this correction becomes imperative.

If A denotes also the point on wire A where the center of the Sun crosses, and B is the corresponding point on wire B, the times of the transit of the center are:

$$\text{at } A \text{ - - - - - } \tfrac{1}{2}(A_1 + A_2),$$
$$\text{at } B \text{ - - - - - } \tfrac{1}{2}(B_1 + B_2),$$

and the interval is:

$$AB = \tfrac{1}{2}(B_1 + B_2) - \tfrac{1}{2}(A_1 + A_2).$$

The time of transit of the center over the normal to the Sun's path through the intersection of the cross-wires is:

$$T = \tfrac{1}{2}(A_1 + A_2) + AB \sin^{2}\alpha.$$

[1] R. C. Carrington, *M. N.*, 14, 155, 1854.

[2] In the original, α was supposed to be measured from the east and west line, giving s the opposite sign.

The diameter of the Sun will be

$$2R = 15 \cdot \sin p\,(B_2 - B_1)\sin \alpha,$$

or if wire B make an angle with wire A, which is not exactly 90°, but 90° + θ, the Sun's diameter is

$$2R = 15 \cdot \sin p\,(B_2 - B_1)\sin \alpha \cdot (1 \pm \tfrac{1}{2}\sin \theta),$$

the sign of $\frac{1}{2}\sin\theta$ changing if the cross-wires are rotated through 90°, thus interchanging wires A and B. The angle θ may evidently be determined by making this interchange.

If D is the distance from the intersection at which the center crosses the normal to the Sun's path,

$$D = AB \cdot \cos \alpha \cdot \sin \alpha.$$

T and D being the normal interval and distance for the Sun's center, while t and d are the same for a Sun-spot, the spot's coördinates relatively to the center are:

In right ascension $15 \cdot \sin p\,(t - T)$,
In north polar distance $15 \cdot \sin p\,(d - D)$,

all quantities being finally expressed in seconds of arc. These values may be transformed into polar coördinates. As in the case of micrometric measures they require no correction for distortion.

For a more complete elucidation of these formulas, the original paper by Carrington, already cited, should be consulted.

In his larger work[1] Mr. Carrington deprecates the establishment of an east and west line on a Sun-drawing by allowing a spot to run across with telescope fixed, noting its path, and undoubtedly the method described by him is more accurate; but if care be taken that the spot shall make a central transit, avoiding whatever curvature of path is produced by the distortion of the field, and subsequently applying the correction for the Sun's motion in declination, the trace of a transiting spot is probably as accurate as the other steps in the production of a projection-drawing, at least if nothing better than paper is used in making the sketch.

[1] R. C. Carrington, *Observations of the Spots on the Sun from November 9, 1853 to March 24, 1861, made at Redhill.* London, 1863.

One especial advantage in the employment of the diagonal transit wires, as advocated by Carrington, is that the chromatic separation of images acts in the direction of the radial cross-wires, while the contacts observed are at right angles to the same direction. Errors due to imperfect achromatism are therefore minimized. If the transits are repeated several times and their mean values taken, the irregularities of measured positions due to the almost universal diurnal atmospheric tremor will be largely eliminated. Accuracy in hand drawings is greatly hindered by tremor, and to such an extent that little advantage is gained by magnifying the image above a diameter of six or eight inches, although but for this trouble there should be increased precision with larger magnification. Instantaneous photography and the method of repeated transits will overcome the difficulty to a great extent.

Carrington's method does not give good results when a spot is near the limb. The micrometric measurement of the spot's distance from the limb ($R-r$), as described by Secchi in his work on the Sun, is then to be preferred. In this case no correction for distortion is needed, but to preserve the micrometer from injury and loss of accuracy through changes of dimension by the heat, special holders and screens are needed, and the wires are preferably made of fine platinum, since if of combustible material the full aperture can scarcely be used. "As it is impossible to know the direction of the center exactly, we dispose the micrometer in such a way that one of its wires may be perpendicular, the other tangent to the limb. It is better that the second [parallel] wire should encroach a little upon the disk [in the preliminary adjustment]. We shall then be able to judge by the equality of its two segments when the recticule is properly placed."[1]

The photographic method, employing a horizontal telescope and heliostat, permits the very exact orientation of the image by the simultaneous photographing of the shadow of a plumb-line of fine wire, suspended in front of the photographic plate and

[1] A. SECCHI, *Le Soleil*, p. 83, Paris, 1870.

almost in contact with it. The position of the north and south line on the image follows immediately by the application of the usual formulas for the parallactic angle.

While photographs are to be preferred, the comparison of the results from carefully made drawings by Rev. S. J. Perry[1] shows that the drawings are not very far behind, and for training the observing powers of the student they are indispensable.

All that precedes relates to the determination of positions in a solar image in plane polar coördinates. The transformation of these coördinates will be considered in another article.

LADD OBSERVATORY,
Providence, R. I., July 1897.

[1] S. J. PERRY, "Photographs and Drawings of the Sun," *Mem. R. A. S.*, **49**, 273, 1889.

Minor Contributions and Notes.

HARVARD COLLEGE OBSERVATORY. CIRCULAR NO. 18.

VARIABLE STAR CLUSTERS.

Announcement was made in *Circular* No. 2 of the discovery by Professor Solon I. Bailey of numerous variable stars in certain globular stellar clusters, and their absence in other objects which apparently belong to the same class. Since then he has found many more of these variables, so that their total number, including a few found here, is now 310, distributed as follows:—In *N. G. C.* 104 (47 Tucanae), 6; in 362, 8; in 1904,1; in 5139 (ω Centauri), 60; in 5272 (Messier 3), 113; in 5904 (Messier 5), 63; in 5986, 1; in 6254, 1; in 6266, 9; in 6626, 3; in 6656, 5; in 6723, 2; in 6752, 1; in 7078, 27; 7089, 8; and in 7099, 2. In the greater portion of these clusters about 1000 stars were examined. In Messier 3 about one-ninth of the stars are variable, while in others like *N. G. C.* 6205 (the great cluster in Hercules), out of nearly 2000 stars not a single variable has been found. The positions of 62 of the stars in Messier 5 are given in the *Harvard Observatory Annals*, XXVI, 243, 246. The light curve and period of one of them are given in the *A. N.*, **140**, 285.

SOUTHERN DOUBLE STARS.

A distinguishing feature of the climate of Arequipa is the great steadiness of the air. The value of this location as an observing station is largely due to this fact. Good definition, under high powers, is obtained there on many more nights than in Europe, or in the United States, where nine-tenths of the observatories of the world are at present located. A search for close double stars may, therefore, be advantageously made at Arequipa, and accordingly, in 1891 all the stars of the sixth magnitude and brighter, south of declination $-30°$, were examined for close companions. The stars in one quarter of the region, and included between 12^h and 18^h of right ascension were examined by Professor William H. Pickering, and the remaining three-quarters by Professor Solon I. Bailey. The instrument used was the 13-inch Boyden telescope. A power of 450 was ordinarily employed.

The stars whose numbers in the Argentine General Catalogue are given below were found to have companions whose distances were estimated not to exceed 30″. Stars already announced as double in the catalogues of Herschel and Russell are not included. When two or more companions were noted the letter T is inserted after the number:—

A. G. C. 451 T, 480 T, 684, 702, 780, 1024 T, 1197, 2412, 2992, 3487 T, 4368, 4395, 4845, 5295, 6494, 6633, 7914, 7990 T, 8093 T, 9234, 10188, 10335, 10496, 11097, 11712, 11727, 11887, 12035, 12101, 12180, 13135, 15795, 16200, 16541 T, 16612 T, 16690 T, 16793, 16845, 16992, 17403, 17440, 17504, 17541 T, 17572, 17907, 17936, 18174 T, 18492 T, 18700, 18773, 18863 T, 18931 T, 18980, 19129, 19273 T, 19280, 19295, 19540 T, 19578 T, 19597 T, 19679, 19697, 19741, 19746, 19873, 19916, 19934 T, 19980, 19988, 20049, 20170, 20203, 20444 T, 20466 T, 20649, 20695, 20806 T, 20811, 20861, 20909 T, 21078 T, 21153, 21177, 21319 T, 21374, 21421 T, 21499 T, 21559, 21694 T, 21767, 21828, 21887, 21921, 21974, 21995, 22124, 22159, 22534 T, 22582, 22598, 22604, 22949, 22970, 23018, 23035 T, 23098, 23126 T, 23358, 23397 T, 23448 T, 23486, 23515 T, 23549 T, 23597 T, 23603 T, 23785 T, 23973, 24148, 24182, 24218, 24407 T, 24483, 24557, 24570, 24624, 24703, 24888, 25137, 25259, 25527, 25548, 26026 T, 26041, 26287, 27354, 27909, 28851, 29314, 29368, 30425, 30814, and 32446.

SPECTRUM OF ζ PUPPIS.

In *Circular* No. 16 it was shown that a line having wave-length 5413.9 probably exists in the spectrum of this star. This line is clearly visible on three photographs of this star taken in Arequipa on isochromatic plates. EDWARD C. PICKERING.

JULY 29, 1897.

NOTE ON THE DISTORTION DUE TO THE LENS IN A PROJECTION DRAWING.

THE following observations were made in connection with a series of projection-drawings of the Sun with a 12-inch equatorial. An image of the Sun, of three inches radius, was allowed to pass over twenty-five parallel lines, one-quarter inch apart, carefully adjusted perpendicular to the direction of the Sun's motion. The times of passage over each line of both limbs and a few small spots were recorded by the chronograph. The time-intervals from the center

were found, and the correction for the Sun's motion applied. The means of the time-intervals from the center to the limb were taken and divided in succession by the largest, *i. e.*, by the time corresponding to the passage of the radius. This gives the percentage of times, and these are compared with the percentage which would be found if there were no distortion. This latter is the same as the percentage of linear intervals, and is found by dividing the distances from the center by the radius. The differences between these two percentages give the corrections to $\frac{r}{R}$ in Carrington's notation, where r is the distance of the spot from the center of the projected image, and R is the radius of the projection.

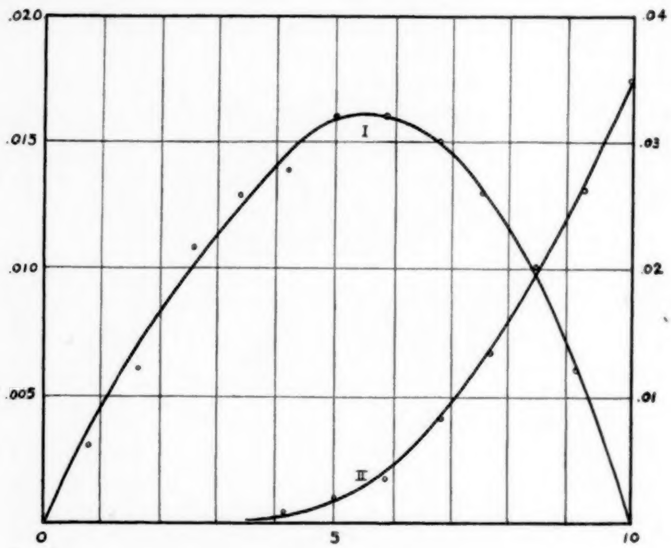


FIG. 1.

Curve I, Fig. 1, has for ordinates these corrections and for abscissæ the distances from the center.

To find the correction in inches to any distance on the projection measured from the center, the mean intervals of times, corresponding to the same radius as found from the transit observations across the parallel lines, were compared with the intervals corresponding to an undistorted image. These were found by considering the center of

Line	Transit intervals										Mean	Sun's motion	Corrected mean
	Limb 1	Limb 2	Limb 1	Limb 2	Limb 1	Limb 2	Limb 1	Limb 2	Spot	Spot			
	s	s	s	s	s	s	s	s	s	s	s	s	
1..........	66.65	66.03	66.65	65.84	66.76	65.88	66.91	65.70	66.30	66.71	66.34	0	66.34
2..........	61.60	60.78	61.55	60.69	61.58	61.38	61.61	61.50	61.35	61.61	61.36	—.01	61.35
3..........	56.25	55.95	55.85	55.94	56.08	55.93	56.26	55.85	55.85	56.36	56.03	—.03	56.00
4..........	50.83	50.65	50.75	50.74	50.98	50.78	50.81	50.80	50.85	51.11	50.83	—.04	50.79
5..........	45.25	45.35	45.25	45.14	45.38	45.33	45.31	45.40	45.50	45.51	45.34	—.06	45.28
6..........	39.99	40.00	40.00	40.04	40.18	39.93	39.86	39.90	39.80	39.96	39.96	—.07	39.89
7..........	34.54	34.05	34.35	34.34	34.33	34.43	34.46	34.40	34.20	34.51	34.42	—.08	34.34
8..........	28.75	28.88	28.85	28.79	28.73	28.93	28.86	28.90	28.85	28.81	28.84	—.10	28.74
9..........	22.90	23.05	23.15	23.14	23.03	23.28	23.01	23.30	23.10	23.11	23.11	—.12	22.99
10..........	17.45	17.55	17.35	17.64	17.58	17.58	17.61	17.55	17.50	17.26	17.51	—.13	17.38
11..........	11.40	11.82	11.75	11.89	11.68	11.78	11.71	11.70	11.80	11.61	11.71	—.15	11.56
12..........	5.93	5.80	6.05	6.12	5.98	6.18	6.21	6.10	6.05	6.26	6.07	—.16	5.91
13..........	.05	.55	.05	.44	.08	.38	.31	.20	.30	.16	.25	—.18	.07
14..........	5.86	5.25	5.85	5.86	5.72	5.72	5.79	5.75	5.90	5.69	5.74	—.19	5.55
15..........	11.65	11.70	11.65	11.61	11.72	11.52	11.39	11.80	11.30	11.49	11.58	—.21	11.37
16..........	17.65	17.65	17.37	17.46	17.57	17.52	17.79	17.50	17.10	17.49	17.51	—.22	17.29
17..........	23.05	23.10	23.20	22.86	23.27	22.82	23.19	23.00	23.20	23.19	23.09	—.24	22.85
18	28.61	28.70	28.75	28.46	28.72	28.42	28.79	28.60	28.72	28.99	28.71	—.25	28.46
19..........	34.45	34.10	34.25	34.36	34.52	34.42	34.39	34.45	34.20	34.59	34.37	—.27	34.10
20..........	39.94	40.00	39.77	40.06	39.97	39.77	39.99	39.90	40.15	39.94	39.95	—.28	39.67
21..........	45.40	45.35	45.40	45.46	45.72	45.32	45.79	45.45	45.45	45.49	45.48	—.30	45.18
22..........	50.85	50.50	50.95	50.56	50.72	50.72	50.89	50.75	50.80	51.04	50.78	—.31	50.47
23..........	56.21	56.10	56.05	56.06	56.22	56.32	56.29	56.10	56.15	56.54	56.20	—.33	55.87
24..........	61.46	61.20	61.65	61.21	61.67	61.37	61.79	61.35	61.50	61.69	61.49	—.34	61.15
25..........	66.52	66.70	66.55	66.91	66.62	66.52	66.79	66.80	66.70	66.79	66.69	—.36	66.33

Mean of intervals from center	Percentage of times	Percentage of distances	Correction to $\frac{r}{R}$	Distance from center	Mean of intervals from center	Corresponding intervals taking 5.73 as unit for ¼ inch.	Differences	Corresponding value in inches. Correction to measured distance from center
s				in.	s	s	s	in.
66.34	1.00	1.00	+0	3.00	66.34	68.76	2.42	−.106
61 25	.923	.917	+.006	2.75	61.25	63.03	1.78	−.077
55.94	.843	.833	+.010	2.50	55.94	57.30	1.36	−.059
50.63	.763	.750	+.013	2.25	50.63	51.57	.94	−.041
45.23	.682	.667	+.015	2.00	45.23	45.84	.61	−.027
39.78	.599	.583	+.016	1.75	39.78	40.11	.23	−.010
34.22	.516	.500	+.016	1.50	34.22	34.38	.16	−.007
28.60	.431	.417	+.014	1.25	28.60	28.65	.05	−.002
22.92	.346	.333	+.013	1.00	22.92	22.92	0	0
17.34	.261	.250	+.011	.75	17.34	17.19	[15]	[007]
11.46	.173	.167	+.006	.50	11.46	11.46	0	0
5.73	.086	.083	+.003	.25	5.73	5.73	0	0
	0	0	+0	.0				
			See curve No. I					See curve No. II

the image as undistorted, and taking the time of passage over the middle space as the unit of time for each space. The differences between these times were found and reduced to inches by dividing by four times the central unit, since this unit corresponds to one-fourth of an inch.

Curve II, Fig. 1, has for ordinates these corrections, and for abscissæ the distances from the center.

The results tabulated are self-explanatory.

FRED SLOCUM.

PROVIDENCE, R. I.,
May 1897.

THE YERKES OBSERVATORY OF THE UNIVERSITY OF CHICAGO, BULLETIN NO. 3.

DEDICATION OF THE YERKES OBSERVATORY.

THE following additions should be made to the programme of the dedication of the Yerkes Observatory published in the last number of the ASTROPHYSICAL JOURNAL.

The principal scientific address to be made at the Observatory on

October 21, in connection with the formal presentation of the building and instruments to the University of Chicago, will be delivered by Professor James E. Keeler, Sc. D., Director of the Allegheny Observatory. The subject of the address is "The Importance of Astrophysical Research, and the Relation of Astrophysics to other Physical Sciences."

The subject of Professor Newcomb's address, which will be delivered at the University of Chicago on the following day, is "Aspects of Modern Astronomy."

On Monday evening, October 18, Professor S. W. Burnham will show selected double stars with the 40-inch Yerkes telescope. This part of the programme, in common with others involving telescopic observations, is, of course, subject to change in the event of unfavorable weather.

Professor Simon Newcomb will speak at the conferences on a subject to be announced later.

Professor Comstock has changed the title of his paper to "Researches at the Washburn Observatory." It will be presented on Tuesday, Oct. 19, instead of Wednesday, as previously announced. Professor Comstock will exhibit a new form of double-image micrometer.

The title of Dr. Laves second paper has been changed to "Researches on Planet 3.34."

GEORGE E. HALE.

YERKES OBSERVATORY,
September 1897.

Recent Publications.

A list of the titles of recent publications on astrophysical and allied subjects will be printed in each number of the Astrophysical Journal. In order that these bibliographies may be as complete as possible, authors are requested to send copies of their papers to both Editors. For convenience of reference, the titles are classified in thirteen sections.

1. The Sun.

Hadden, D. E. Solar Observations, 1895-6. Pub. A. S. P. **9**, 77-85, 1897.

Hale, George E. Note on the Relative Frequency of the H and K Lines in the Spectrum of the Chromosphere. Ap. J. **6**, 157-158, 1897.

Lockyer, J. Norman. The Approaching Total Eclipse of the Sun. Nat. **56**, 154-157, 175-178, 1897.

MacDowall, Alex. B. Sonnenflecke und Luft-Temperature. Meteorolog. Zeit. **14**, 278, 1897.

Mascari, A. Sulla frequenza e distribuzione in latitudine delle macchie solari osservate all' Osservatorio di Catania nel 1895. Mem. Spettr. Ital. **26**, 45-60, 1897.

Maunder, E. W. Total Solar Eclipse, January 22, 1898. English Preparations. Pub. A. S. P. **9**, 131-134, 1897.

Perrine, C. D. Some Recent Sun-spots. Pub. A. S. P. **8**, 225-227, 1896.

Quimby, A. W. Sun-spot Observations. A. J. No. 407, **17**, 183-184. 1897.

Riccò, A. On the Level of Sun-spots and the Cause of their Darkness. Ap. J. **6**, 91-94, 1897.

Rizzo, G. B. Misure assolute del calore solare fatte alla capanna "Regina Margherita" nel Monte Rosa. Mem. Spettr. Ital. **26**, 79-93, 1897.

Rydzewski, Alexander. Total Eclipse of August 9, 1896. Pub. A. S. P. **8**, 297-303, 1896.

Tacchini, P. Sulla distribuzione in latitudine dei fenomeni solari osservati nel 2° trimestre del 1897 al Regio Osservatorio del Collegio Romano. Mem. Spettr. Ital. **26**, 94-99, 1897.

TACCHINI, P. Macchie e facole solare osservate al R. Osservatorio del Collegio Romano durante il 2° trimestre del 1897. Mem. Spettr. Ital. **26**, 65–68, 1897.

TACCHINI, P. Sulle protuberanze solari osservate al R. Osservatorio del Collegio Romano durante il 2° trimestre del 1897. Mem. Spettr. Ital. **26**, 69–70, 1897.

TACCHINI e RICCÒ. Immagini spettroscopische del bordo solare desegnate a Catania e Roma nei mesi di agosto e settembre del 1895. Tavola CCCXXXIII. Mem. Spettr. Ital. **26**, 1897.

TACCHINI e RICCÒ. Immagini spettrali del bordo solare osservate a Catania e Roma nei mesi di novembre e dicembre, 1895. Tavola CCCXXXV. Mem. Spettr. Ital. **26**, 1897.

YOUNG, C. A. On the Reversing Stratum and its Spectrum, and on the Spectrum of the Corona. Ap. J. **6**, 155–157, 1897.

3. STARS AND STELLAR PHOTOMETRY.

FENET, LÉON. L'Amas du Toucan. Bull. Soc. Astr. France, 257–261, July, 1897.

FOWLER, A. A New Classification of Stellar Spectra (Review). Nat. **56**, 206–208, 1897.

O'HALLORAN, ROSE. Observations of Variable Stars. Pub. A. S. P. **8**, 254, 1896.

O'HALLORAN, ROSE. Maximum of o Ceti, 1896–7. Pub. A. S. P. **9**, 86–109, 1897.

HUSSEY, WILLIAM J. Nova Centauri, etc. Pub. A. S. P. **8**, 220–222, 1896.

LOCKYER, J. NORMAN. The Chemistry of the Hottest Stars. Nat. **56**, 91–92, 1897.

PARKHURST, HENRY M. Notes on Variable Stars — No. 18. A. J. No. 403, **17**, 147–149, 1897.

PARKHURST, J. A. Maxima and Minima of Long-Period Variables. A. J. No. 405, **17**, 167–168, 1897.

PARKHURST, J. A. Maximum of 2815 U Geminorum. A. J. No. 407, **17**, 182, 1897.

4. STELLAR SPECTRA, DISPLACEMENTS OF LINES AND MOTIONS IN THE LINE OF SIGHT.

GRUS, G. Spectroskopische Beobachtungen einiger Sterne. Prag, 1897.

MONCK, W. H. S. The Spectra and Proper Motion of Stars. Pub. A. S. P. **9**, 123–128, 1897.

5. PLANETS, SATELLITES AND THEIR SPECTRA.

BRENNER, LEO. Observations de Saturne. Bull. Soc. Astr. France 326–327, August 1897.

CAMPBELL, W. W. Review of Mr. Lowell's Book on Mars. Pub. A. S. P. **8**, 207–220, 1896.

CAMPBELL, W. W. Recent Observations of the Spectrum of Mars. Pub. A. S. P. **9**, 109–112, 1897.

DOUGLASS, A. E. Nuages sur Mars. Bull. Soc. Astr. France, 290–292, July 1897.

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GRIFFITHS, HENRY F. La Planète Mars durant la dernière opposition. Bull. Soc. Belge d'Astr. **2**, 178, 1897.

MOYE, M. Jupiter en 1897. Bull. Soc. Belge d'Astr. **2**, 215–216, 1897.

PEYRA, D. Note sopra Marte Opposizione, 1896–7. Mem. Spettr. Ital. **26**, 61–64, 1897.

PICKERING, W. H. La Météorologie de Mars. Bull. Soc. Belge d'Astr. **2**, 221–224, 1897.

QUENISSET, F. Observations de Jupiter. Bull. Soc. Ast. France, 318–323, August 1897.

6. COMETS, METEORS AND THEIR SPECTRA.

BARONE, G. Les Bielides de 1895. Bull. Soc. Belge d'Astr. **2**, 207–209, 1897.

EDITOR BULL. SOC. ASTR. FRANCE. Étoiles filantes d'août 1896. Bull. Soc. Astr. France, 328–329, August 1897.

HERSCHEL, A. S. Bolides des 23 décembre 1895 et 16 decembre 1896. Bull. Soc. Astr. France, 292–294, July 1897.

7. NEBULÆ AND THEIR SPECTRA.

ROBERTS, ISAAC. Photograph of the Nebula Herschel V 14 Cygni Knowledge **20**, 218, 1897.

SCHEINER, J. Spectroscopic Observations of Nebulæ made at Mount Hamilton by J. E. Keeler (Review). V. J. S. Astr. Gesell. **32**, 42–52, 1897.

SWIFT, LEWIS. Catalogue No. 1 of Nebulæ Discovered at the Lowe Observatory, California. Pub. A. S. P. **9**, 304–306, 1896.

8. TERRESTRIAL PHYSICS.

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EDITOR TERRESTRIAL MAGNETISM. Note: The Elementary Pulsations of the Earth's Magnetism. Terr. Mag. **2**, 84, 1897.

FLEMING, J. A. The Earth, a Great Magnet. Terr. Mag. **2**, 45–60, 1897.

SCHEINER, J. Die Helmholtzschen Untersuchungen über Bewegungsvorgänge in unserer Atmosphäre. Himmel und Erde, **9**, 433–451, 1897.

DE TILLO, A. Isanomales et variations séculaires des composantes Y et Z de la force magnétique horizontale pour l'époque 1857. Terr. Mag. **1**, 163–168, 1896.

9. EXPERIMENTAL AND THEORETICAL PHYSICS.

AMES, J. S. and W. J. HUMPHREYS. Note on the Effect of Pressure upon the Series in the Spectrum of an Element. Johns Hopkins University Circulars, June 1897, Phil. Mag. **44**, 119–121, 1897.

ANDERSON, A. On the maximum deviation of a ray of light by a prism. Proc. Cambridge Phil. Soc, **9**, 195–197, 1896.

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CUSACK, RALPH. Effect of Change of Temperature on Phosphorescent Substances. Nat. **56**, 102, 1897.

DÉCOMBE, L. Sur la résonance multiple des ondulations électriques. Arch. de Genève, **4**, 30–38, 1897.

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MENDENHALL, C. E. and SAUNDERS, F. A. Preliminary Note on the Energy Spectrum of a Black Body. Johns Hopkins University Circulars, June 1897. Phil. Mag. **44**, 136, 1897.

PAUER, J. Absorption ultravioletter Strahlen durch Dämpfe und Flüssigkeiten. Wied. Ann. **61**, 363–379, 1897.

PERRIN, JEAN. Décharge par les rayons X; influence de la pression et de la température. Séances Soc. Franc. Phys. 288, 1896.

PRESTON, THOMAS. On a Supposed Proof of a Theorem in Wave-Motion. Phil. Mag, **43**, 458–460, 1897.

STOKES. On the Nature of the Röntgen Rays. Proc. Cambridge Phil. Soc. **9**, 215–216, 1896.

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THIELE, T. N. On the Law of Spectral Series. Ap. J. **6,** 65–76, 1897.

WADSWORTH, F. L. O. On the Conditions which Determine the Ultimate Optical Efficiency of Methods for Observing Small Rotations, and on a Simple Method of Doubling the Accuracy of the Mirror and Scale Method. Phil. Mag. **44,** 83–97, 1897.

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WIND, C. H. Over de dispersie der magnetische draaüng van het polarisatievlak. Verslagen K. Akad. Wetens. Amsterdam, **6,** 92–94, 1897.

WOOD, R. W. A New Form of Cathode Discharge and the Production of X-Rays, together with some Notes on Diffraction. Phys. Rev. **5,** 1–10, 1897.

ZEEMAN, P. Doublets and Triplets in the Spectrum produced by External Magnetic Forces. Phil. Mag. **44,** 55–60, 1897.

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10. THE SPECTRA OF THE ELEMENTS.

HUGGINS, SIR WILLIAM and LADY HUGGINS. On the relative Behavior of the H and K Lines of the Spectrum of Calcium. Ap. J. **6,** 77–86, 1897.

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11. PHOTOGRAPHY.

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LIVEING. On Photographing the whole length of a Spectrum at once. Proc. Cambridge Phil. Soc. **9,** 141–142, 1896.

NAEGAMVALA, K. D. On a Method of Reproducing Astronomical Photographs. Nat. **56,** 153, 1897.

WADSWORTH, F. L. O. Astronomical Photography.—I. The Conditions which Determine the Limiting Time of Exposure. Knowledge, **20,** 193–195, 1897.

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12. INSTRUMENTS AND APPARATUS.

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LORD, H. C. The New Photographic Correcting Lens of the Emerson McMillin Observatory. Ap. J. **6**, 87–90, 1897.

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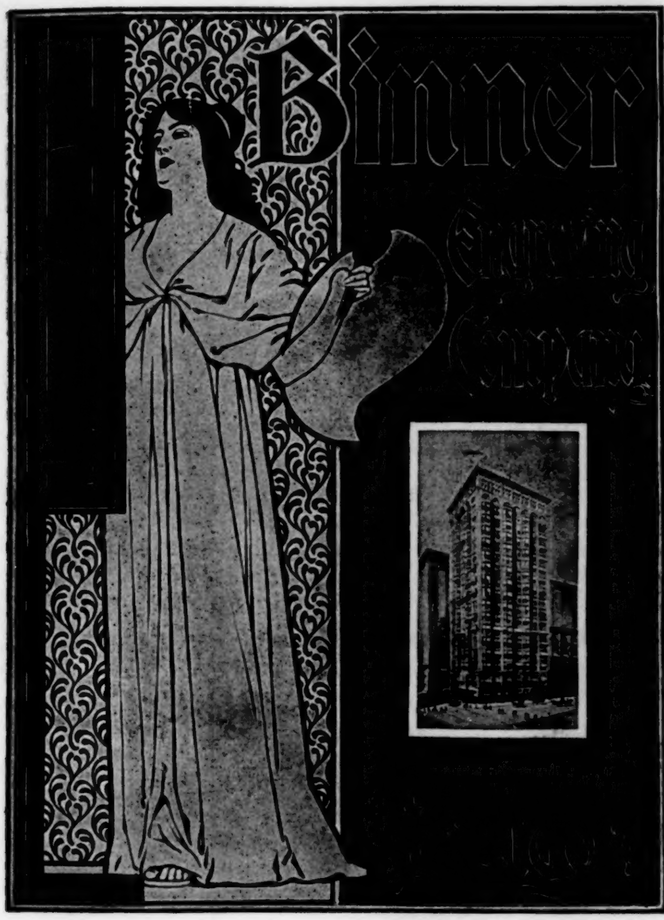

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